Theories of Political Coalitions

Joris Alberdingk Thijm

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Instituto de Ciências Sociais, Universidade de Lisboa

Thesis advisors: Dr. Marcelo Camerlo, Dr. Andrés Malamud

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Introduction

The aim of this chapter is to discuss a selection of the most important theories of coalition formation that have emerged in the literature since the mid-20th century. This exercise will ultimately inform the formulation of an original theoretical model of government coalitions in presidential systems in a subsequent chapter. Four different but interconnected literatures can inform our attempt to build such model. These are: (1) the fundamental game theoretical literature and the two applied literatures inspired by it, namely (2) the literature on government coalitions in parliamentary systems, which is mainly centered on Europe, and (3) the social choice literature on legislative coalitions, which is mostly based on the US context. Aside from those three literatures, there is (4) the younger literature on government coalition formation and management in multiparty presidential systems, which is mainly empirical in origin. This chapter is organized into four main sections, one for each aforementioned literature. The focus will be on theories and theoretical arguments; we do not concern ourselves here with the findings of empirical studies that operationalize and test these theories. Furthermore, as explaining oversized coalitions in presidential systems is one of the main objectives of this thesis, the focus will be of theories and models which have something to say about coalition size.

General Coalition Theory

Coalitions and Game Theory

Game theory is not simply a methodology to study coalitions, but rather, coalitions are a fundamental aspect of game theory itself. Von Neumann & Morgenstern's *Theory of Games and Economic Behavior* (1944)[1] is generally considered to be the founding work of the field of game theory. Rather than reproducing the complete axiomatic definition of the concept of a game according to Von Neumann & Morgenstern (henceforth referred to as N-M) here, for the moment it suffices to say that games, in the commonly understood sense of the word, can be subdivided into a number of categories on the basis of a number of dimensions. One of these dimensions is the number *n* of players. Games can have any number of players, and this property is customarily adjectivized in the following way: games with one player are called one-person games, games with two players two-person games, and more generally, games with *n* players are called *n*-person games (Von Neumann & Morgenstern, 1953, p. 47). Another dimension is the sum of the payments received by all players at the end of the game (p. 46). If this sum equals zero, "players pay only to each other, and […] no production or destruction of

[1] In this thesis, the third edition of this work, published in 1953, was consulted. Henceforth the citation Von Neumann & Morgenstern (1953) will be used.

goods is involved" (ibid.). Thus, the gains of the winners are exactly equal to the losses of the losers. Games where this condition holds are called zero-sum games, and those where it does not hold are called non-zero-sum games (p. 47).

Coalitions as a formal concept first appear in N-M's book at the beginning of Chapter 5, which is titled *Zero-Sum Three-Person Games*. Hence, when games are ordered by their complexity from least complex to most complex, which they are in the ordering of the book's chapters (p. 85), the zero-sum three-person game is the least complex game in which coalitions are relevant. The reason for this can be demonstrated easily. I will not elaborate on the changes occurring when moving from one-person games (in which the zero-sum condition is void (p. 86)) to two-person zero-sum games for the sake of brevity, as one-person games are a special case with little relevance for our purposes. However, when moving from the two-person zero-sum game to the three-person zero-sum game, something fundamental changes. While two-person games are characterized by an "absolute antagonism of interests," meaning that every gain for one player entails a loss for the other player and vice-versa, this is not necessarily the case in three-person games, where the gain of one player entails the loss of at least one of his two opponents, but not necessarily of both (p. 220). Hence, in these games, a "parallelism of interests" may occasionally arise between two, but never all, of the players (ibid.). This incidental parallelism of interests in turn makes cooperation desirable whenever it occurs, and can lead to agreements being formed between pairs of players (p. 221). We are now talking about coalitions.

But this does not mean that coalitions are always the best option in all 3-person zero sum games. It still depends on whether a player who cooperates with others can actually get a higher payoff than if he were to play alone. If this is not the case, coalitions have no "raison d´être", that is, they are inessential (pp. 231-232). Games where players playing by themselves can get the same amount that they could obtain by joining any coalition are therefore called *inessential games* (p. 232). On the other hand, in games where a player can obtain more by cooperating than by playing alone, there exists a "definite inducement to form coalitions", and so these games are called *essential games* (ibid.).

Essentiality as a property is relevant not only in 3-person zero-sum games, but can be applied to all games (p. 232). Within the essential games, N-M identify a category to which they attach special importance: the *simple games*. Simple games are games where all possible coalitions are either winning or losing, that is, no coalitions can exist that are both winning and losing or both not winning and not losing (p. 428). For instance, in the so-called *direct majority game*, where any coalition consisting of a majority of the players wins, simplicity means that there can be no coalition containing exactly half of the players (p. 431). That is, the total number of players must be odd (ibid.). But this is only true if players are treated as having equal weights. If the weights are allowed to vary, games with an even number of players can also be simple, as long as no coalition *S* has the same combined weight as its complement -*S*[2], or no coalition *S* has a weight that is exactly equal to half the combined weight of all players (pp. 432-33). This latter type of simple game with weighted players is called a *weighted majority game* (p. 433).

Before moving on to the next section, three more interrelated concepts must be introduced: *imputation*, *domination* and *solution*. An imputation, in simple terms, refers to the distribution of the ‘proceeds’ of a game among its participants (p. 34). Recall that in zero sum games the total proceeds are always equal to zero (p. 34, footnote 2), so that each of the losers’ ‘share’ of the proceeds will be negative, whereas each of the winners’ share will be positive, and the total losses of the losers cancel out the total gains of the winners. Of course, there are many ways in which the proceeds of a game can be distributed among its participants, meaning there are many possible imputations, some more advantageous for (a majority of) the participants than others. Note that the actual distribution of payoffs among the members of a given coalition is not only determined by the rules of the game, but also depends on the alternative coalitions that each of them could have joined and the payoffs they could have secured in so doing (p. 227). Hence, under some circumstances, players have to pay compensations to their prospective partners, the amount of which then depends on the alternatives available to the players (ibid.).

When comparing these imputations in terms of their advantageousness, the concept of domination enters the stage. An imputation *x* is said to dominate another imputation *y* when “a sufficient number of participants [of a game] prefer in their own interest *x* to *y* and are convinced or can be convinced of the possibility of obtaining the advantage of *x*” (p. 37). Note that this description suggests that for one imputation to dominate another, the latter does not have to be more advantageous for each individual participant, but only for a ‘sufficient’ number of them, such that this group of participants is able to “enforce [its] preferences” (p. 38). A group that is able to do so is called the *effective set* for the domination of *x* over *y* (ibid.). However, the effective set for the domination of *x* over *y* may be different (“disjunct”, in the words of N-M) from the effective set for the domination of *y* over a third imputation *z*, and therefore, nothing can be said about the relationship between *x* and *z* (ibid.). It may even be the case that *z* dominates *x* through yet another effective set, disjunct from the two aforementioned ones (ibid.). This relationship, whereby *x* dominates *y* and *y* dominates *z* but *x* does not necessarily dominate z is reminiscent of the game of rock paper scissors and is called *intransitivity* (pp. 38-39). It is an important property of the concept of domination (ibid.).

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[2] The complement -*S* of a coalition *S* is equal to the set of players that are not part of *S* (p. 62). So in a game with players *i*, *j* and *k*, if players *I* and *j* form a coalition *S*, the only member of its complement -*S* is *k*.

These concepts, imputation and domination, are key elements of the definition of the third and final important concept: solution. A solution, according to N-M, is a “set of rules for each participant which tell him how to behave in every situation which might conceivable arise” (p. 31). In game-theoretic terms, then, a solution consists of a “system of imputations possessing in its entirety some kind of balance and stability” (p. 36). A solution may consist of a single imputation that dominates all others (p. 37), but in situations characterized by intransitivity, such as the three-person game, no single imputation dominates all others, and so they require a different kind of solution (p. 39). The solution for these situations put forth by N-M is a set of imputations (p. 39), such that “no [imputation] *y* contained in [the set of imputations] *S* is dominated by an [imputation] *x* contained in *S*” and “every *y* not contained in *S* is dominated by some *x* contained in *S*” (p. 40). Put differently, the solution is the set of imputations that do not dominate one another (ibid.). Some of the imputations that are not part of the solution may dominate some other imputation belonging to the solution, but each of the former imputations will always be dominated by some third imputation contained in the solution. Note that this is a specific solution concept developed by N-M and that other, alternative solution concepts exist, as will become clear in subsequent sections. The types of coalitions predicted by the game theoretical models discussed in this chapter are the solutions to the games these models describe.

The Minimal Winning Coalition

The concept of the *minimal winning coalition* is often attributed to Riker (1962), but actually, N-M (1953) coined this term and defined the concept it refers to. Minimal winning coalitions, denoted by W^m, is are those winning coalitions *W* in which “no participant can be spared”, that is, no participant can be removed without the coalition becoming a losing coalition *L* (p. 430). Formally, then, if we let *W* denote the set of winning coalitions, the set of minimal winning coalitions W^m consists of “those elements of *S* of *W* of which no proper subset belongs to *W*” (ibid.). Minimal willing coalitions are the solution to the simple game described above. That is, none of the imputations corresponding to minimal winning coalitions dominate each other, and all imputations that do not correspond to minimal winning coalitions are dominated by at least one imputation corresponding to a minimal winning coalition, and so the set of imputations corresponding to minimal winning coalitions is the solution to the simple game (pp. 436-42). Put differently, in games where the “sole aim” of players is to build winning coalitions and where coalitions are either winning or losing—meaning no stalemates are possible—“minimal winning coalitions—the S of W^m—are the structures that will form” because “there is no motive for [the members of such coalitions] to admit additional members” (p. 436).

However, that leaves the division of the spoils among the members of these minimal winning coalitions undefined. For instance, even in the 3-person essential zero-sum game, the most basic simple game, N-M find infinite ways of dividing the spoils between any two members of a minimal winning coalition, that is, the solution to that game consists of an "infinite set of imputations" (pp. 288-289).

While N-M invented the minimal winning coalition, it was Riker who became famous for it as he wrote an entire book on the subject. Not satisfied with N-M's solution to the *n*-person games, which are based on the delimitation of admissible imputations and did not result in any unique imputation or corresponding coalition, only sets of imputations and coalitions, Riker argued that coalition structures could be delimited directly, which he attempted to demonstrate (pp. 37-39). Rather than formulating his thesis as a solution to *n*-person games, his aim was simply to derive a 'sociological law', which he named the *size* principle (p. 46). Interestingly, while Riker does cite N-M, he presents the minimal winning coalition—which he calls the minim*um* winning coalition—as if it were of his own invention, without acknowledging N-M's earlier usage of a nearly identical concept under the name minim*al* winning coalition, nor explaining how his concept differs from that of N-M, if at all.

In any case, Riker's argument can be expressed verbally as follows. Starting out from the assumptions of the *n*-person zero-sum game with perfect information and with side-payments[3] being allowed (p. 39), he makes a number of statements which are part assumption, part proposition. (1) "Let the sum of what the winners gain be equal to the sum of what the losers lose" (the zero-sum condition); (2) "When a coalition includes everybody, the winners gain nothing simply because there are no losers"; (3) "The members of a winning coalition have control over additional entries into their coalition"; (4) "The worst a player can do is to enter a coalition of its own", which corresponds to a maximum loss of an amount $-\gamma$ for that player (cf. N-M, p. 248); (5) "A coalition of all but one of the players can win at most the amount γ inasmuch as there is only one loser"; (6) "Let a winning coalition be defined as one which is as large as or larger than some size [m] arbitrarily stated in the rules", the other coalition types being 'losing' (the complement of a winning coalition) or 'blocking' (of size m, so neither winning nor losing and a violation of N-M's simplicity assumption); (7) "If there are n players in the game, a winning coalition of k players, where k is some arbitrary number, can win at most $\gamma(n - k)$", which is a more general version of statement 5; (8) As long as the *characteristic function* $v(S)$[4] of winning coalitions is linearly and inversely related to the number of members

[3] Riker describes side-payments as "private agreements" made by players about the division of payoffs (which N-M call 'proceeds') among them (p. 34). These side-payments seem to be equivalent to the 'compensations' N-M wrote about, which were mentioned in the previous section.

[4] The characteristic function is described by Riker as "the statement of total payment to each coalition possible in the game" (p. 36). Again, this term comes from N-M, who define it as "the value of a play for

of a coalition, which Riker argues is the case for virtually all *n*-person, zero-sum social situations, then (9) “for any coalition larger than minimum, its members can, by ejecting one or more of their members, increase the amount to be divided among them”, and therefore, (10) “among rational players with perfect information, only minimum winning coalitions occur” (pp. 39-46).

The Minimal Size Coalition

When each of the participants in a game has equal weight, minimum winning coalitions will all be of the same size and none of them will have an advantage over any other (Riker, p. 133). However, if participants have different weights, there can be a number of minimum winning coalitions of different sizes (ibid.). And if we recall Riker’s critique of N-M, his ambition was to single out one specific minimum winning coalition that is the best of all, which he calls the *uniquely preferable winning coalition* (p. 129). For a winning coalition to be uniquely preferable, it must meet two criteria: (1) “it has a greater value than any other one possible”, and (2) “it is one in which all participating proto-coalitions can satisfy their initial expectations” (pp. 129-130).

The *proto-coalition* is a concept that can only be explained in relation to Riker’s coalition building game. In contrast to N-M’s models, which are all static (N-M, p. 44), Riker’s model is dynamic (Riker, p. 103), meaning that it treats coalition formation as a process that unfolds over time in a number of stages (p. 105). In the first stage, there are *n* single-member proto-coalitions (ibid.), meaning each participant is playing for him or herself, without cooperating. Whereas a coalition is the end product of coalition-building, the word proto-coalition simply refers to an ‘association’ that the player joins but which does not yet constitute a fully-fledged winning or blocking coalition of size $\geq m$ (pp. 103-104). In the second stage of the game, there is one two-member proto-coalition and $(n - 2)$ single-member proto-coalitions (p. 105). In the last stage, *r*, there is either one winning coalition or two blocking ones (ibid.). In the penultimate or $(r - 1)^{th}$ stage, meaning the one just before the r^{th} stage, a winning coalition can be “created by the union of two proto-coalitions” (ibid.).

According to Riker, coalitions form and grow as the result of “the action of leaders in attracting followers” (p. ibid.). Leaders do this by offering side-payments to potential followers, and these payments can come in the form of any resource which has value to the participants, such as policy promises (ibid.). These resources can already be in the possession of the leaders at the time of negotiation, but they can also come in the form of a promise of a share of the gains that the coalition is expected to win if successful (p. 106). As these resources are scarce, leaders seek

the coalition of all [players] *k* belonging to *S*” (p. 238), a play being a “particular instance at which the game is played […] from beginning to end” (p. 49).

to limit their usage as much as possible, meaning that they will necessarily also limit the number of followers they invite to their proto-coalitions (pp. 106-107). Being aware of the zero-sum nature of the game they are participating in, and knowing they will lose something valuable if the newly formed proto-coalition reaches winning size, those left out of that initial proto-coalition will attempt to form rival proto-coalitions (p. 107). This leads to an i^{th} stage somewhere in between the second stage and the $(r-1)^{th}$ stage, in which there are at least two multimember proto-coalitions (ibid.).

Returning now to the uniquely preferable winning coalition, Riker illustrates under what conditions such a coalition exists using an example[5] in which there are 3 proto-coalitions in the $(r-1)^{th}$ stage, named P^3, Q^3 and R^3, with different weights w, ordered from largest to smallest such that $w(P^3) > w(Q^3) > w(R^3)$ (p. 130). For the time being it is assumed that these proto-coalitions are indivisible units, albeit in reality they are composed of varying numbers of individual players (p. 129). Setting the majority rule m to be equal to a simple majority, meaning $m = (n + 1)/2$ or $m = (n/2) + 1$, and recalling that proto-coalitions are per definition of size $< m$, three different minimal winning coalitions can be formed: $(Q^3 \cup R^3)$, $(P^3 \cup R^3)$ and $(P^3 \cup Q^3)$ (p. 130). By virtue of the size principle, the values of these three minimal winning coalitions are related as follows:

$$\text{If} \begin{Bmatrix} v(Q^3 \cup R^3) = a = -v(P^3) \\ v(P^3 \cup R^3) = b = -v(Q^3) \\ v(P^3 \cup Q^3) = c = -v(R^3) \end{Bmatrix}, \text{then } a > b > c \text{ (p. 131).}$$

Recall that the gains of the winners equal the losses of the losers, so the value of $(Q^3 \cup R^3)$ is equal to the negative value of P^3, and so forth. The greater the proto-coalition that is left out, the greater the gain for the winning coalition, and given that P^3 is the largest proto-coalition, $(Q^3 \cup R^3)$ is the most valuable of all three winning coalitions (ibid.). This satisfies the first condition of the uniquely preferable winning coalition (ibid.). But what about the second condition, that the participants can satisfy their initial expectations? Before answering this question it is necessary to reflect on the meaning of this condition and why it is imposed.

The second condition is imposed so that the excluded proto-coalition cannot bribe one of the members of the winning coalition to defect and join her instead (p. 282). In terms of the above example, P^3 would be willing to accept a payment of zero from either Q^3 or R^3 in order to avoid paying $-a$ (ibid.). In return, Q^3 or R^3 would strike up the entire value of the winning coalition either of them would form with P^3 (p. 131). In the case of $(P^3 \cup R^3)$, R^3 would win

[5] In the *Appendix II* to his book, Riker explores a host of other scenarios with up to 5 proto-coalitions in the $(r-1)^{th}$ stage, several different weight relations among these proto-coalitions as well as different values of m (pp. 279-292). However, the example discussed here suffices to illustrate the logic of Riker's argument.

b and in the case of $(P^3 \cup Q^3)$, Q^3 would win *c* (ibid.). Hence, in order for $(Q^3 \cup R^3)$ to be stable, Q^3 must get a payoff with a value $\geq c$ and R^3 a payoff of $\geq b$. These payoffs are the initial expectations of those two proto-coalitions (ibid.). Whether Q^3 and R^3 obtain their initial expectations depends on the relative sizes of *a*, *b* and *c*, which in turn depends on the shape of the characteristic function (ibid.). If $c < a - b$, then $b > a - c$ (ibid.). Hence, if R^3 gets *b*, there is still $a - b$ left to pay Q^3 with, and Q^3 only expects *c*, which is less than $a - b$ per the aforementioned inequality (ibid.). Hence, under this condition, Q^3 and R^3 obtain their initial expectations and the coalition between them is stable (p. 132). As $(Q^3 \cup R^3)$ now meets both conditions—greatest value and satisfaction of initial expectations—it is a uniquely preferable winning coalition (ibid.). Under different relative sizes of *a*, *b* and *c*, however, it may be that there is no single uniquely preferred winning coalition (p. 285).

De Swaan (1973) criticized Riker's argument on the grounds that the values Riker assigns to the coalitions do not follow automatically from the size principle, unless it is assumed that the value of a coalition is a function of its weight (p. 57). In that case, the value of a coalition is determined by the combined weight of its component proto-coalitions, which is in turn a function of each proto-coalition's number of individual members, each of which possesses one vote (p. 58). If these members are allowed to desert or expel one another from their proto-coalitions, we are now again talking about a game with players of equal weight, meaning that any winning coalition will ultimately be reduced to size *m*, and none of them will be uniquely preferable to any other (ibid.). On the other hand, if coalitions are taken to be insoluble, then they are the de facto actors of the game, and then it does not follow from Riker's assumptions that their value should decrease with their weight (ibid.).

In his discussion on strategy for proto-coalitions in the $(r - 1)^{\text{th}}$ stage, Riker indeed relaxes the insolubility condition (p. 139). Here, proto-coalitions with different sizes have different bargaining positions, which correspond to positions of greater or lesser advantage. More specifically, there is the *uniquely favored proto-coalition*, for which any winning coalition containing such a proto-coalition is more valuable than one not containing it, the *uniquely essential proto-coalition*, which appears in every possible winning coalition when no other proto-coalition does so, and the *strategically weak proto-coalition*, which is not part of the most valuable winning coalition (p. 130). Linking back to the aforementioned example with three proto-coalitions, it can be observed that either coalition containing R^3 is more valuable than the coalition that does not contain it, and so R^3 is in fact uniquely favored. There is no uniquely essential proto-coalition in this example, since there is no single proto-coalition that appears in all three winning coalitions, but there is one strategically weak proto-coalition, P^3, which is the only proto-coalition that does not appear in the most valuable winning coalition $(Q^3 \cup R^3)$.

Funnily enough, the weakest proto-coalition is the largest, which is a rather counter-intuitive result (p. 138) that directly contradicts some of the models that will be discussed subsequently, in which the largest party usually comes out as the one with the best bargaining position.

Anyway, Riker argues that there are two things P^3 can do to improve its position: one is to accept a minimum payoff in a coalition with Q^3 or R^3 (whether this is at all possible depends on the specific interrelationship between *a*, *b* and *c*, as mentioned previously), and the other is to split up and move towards a four- or five-set partition (p. 139), which is possible now that the insolubility condition has been removed. He then goes on to explore a number of different ways in which P^3 can partition itself, and the advantageousness of the positions of the different types of partitions originating from it. However, in spite of this elaborate discussion, Riker, as De Swaan points out, seems to be missing the point that allowing coalitions to split up will, at the end of the day, result in coalitions being formed directly between the individual players, which are of equal weight, and lead to the aforementioned situation in which any minimal winning coalition will be of size *m*.

De Swaan instead points to Gamson (1961) as providing a more consistent explanation of why the minimal size coalition is, in Riker's words, uniquely favored (De Swaan, p. 63). Although Gamson is known for his thesis on the *distribution* of resources within coalitions, this same thesis also has implications for coalition size. In Gamson's own words: "Any participant will expect others to demand from a coalition a share of the payoff proportional to the amount of resources which they contribute to a coalition" (p. 376). It must be noted, however, that this is presented as a hypothesis rather than a solution or a logical implication following from a set of premises. Nevertheless, if we take this not wholly unreasonable statement as an assumption, De Swaan notes, the minimal size coalition becomes the preferred outcome of the participants (p. 63). According to Gamson, under the above assumption, participants will try to maximize their share of resources to the coalition (p. 376). Since their resources, which can be votes in a legislative chamber or industrial capacity in a war, are constant, they will aim to enter the *cheapest winning coalition* possible (ibid.).

There is effectively no difference between this cheapest winning coalition and the minimal size coalition, because Gamson defines coalition winning status in terms of resources, that is, once the coalition possesses a pre-defined share of the total of resources, which he calls the *decision point* and which is analogous to Riker's *m*, the coalition wins and gains control over the decision that is to be made among the participants of the game (p. 374). That decision can then be used to channel a payoff to the winning coalition to the detriment of the opposition (ibid.). Hence, if participants possess different amounts of resources, which Gamson seems to assume, they are in effect weighted, to use the terminology of N-M. The game Gamson describes then seems similar to N-M's weighted majority game. The participants, expecting a payoff proportional to the weight they contribute to the coalition, will then seek to enter the winning coalition with the smallest weight, so that their individual weight as a proportion of the coalition's weight, is maximal. In a parliamentary setting, where the participants are parties made up of legislators each of which possesses one vote, the coalition with the smallest weight is the one with the smallest size, and minimal size coalitions will form.

Theories on Government Coalitions in Parliamentary Systems

The authors discussed up to this point were all agnostic about the institutional context to which their theories applied. Clearly, N-M, Riker and Gamson all had social life in mind, but did not single out any specific context within that broad category. As such, their theories could be applied to any setting, from wars to parliaments, as long as the assumptions were met. The theories discussed from this point are less abstract and more specific about the institutional settings to which they apply. The first set of theories relate to government coalitions in parliamentary systems, and the second set to coalition building in legislatures, inspired on the political system of the US.

Within the first group, the first few theories that will be discussed predict specific coalition shapes to form, usually some variation on the minimal winning coalition, whereas the rest link coalition size to certain independent variables.

Other minimal coalitions

The Minimal Member Coalition

Like Riker and Gamson, Leiserson (1968) was interested in singling out a specific coalition type from N-M's solution to the simple game. In order to do this, he added dynamics ("passage of time"), which introduces the issue of coalition maintenance, as well as 'bargaining uncertainty' to N-M's simple game (p. 775). Together, the addition of these two dimensions resulted in his so-called *bargaining proposition*: "as the number of actors increases there is a tendency for each actor to prefer to form a W^m with as few members as possible" (ibid.). This is, Leiserson argued, because "negotiations and bargaining are easier to complete, and a coalition is easier to hold together [read: maintain], with fewer parties" (ibid.). A minimal winning coalition with fewer members therefore has a bargaining advantage over one with more members, and as a result, the minimal winning coalition with the smallest number of members will form (ibid.). It can once again be objected that if the actors are all of equal weight, any W^m will be a *minimal* *member coalition*[6]. However, when it comes to the actors, Leiserson clearly has the internal factions of political parties (in his case, those of the Japanese Liberal-Democratic Party or LDP, which was, at least until the time of his writing, dominant in the parliament of that country) in mind, the weights of which are determined by the number of individual legislators they are made up of, and so we are back at N-M's weighted majority game (p. 774).

Coalitions with Minimal Ideological Dispersion

Whereas all models discussed up to this point only took into consideration the size, weight and number of members of coalitions, the authors reviewed in this section, Axelrod (1970) and De Swaan (1973), were some of the first to take parties' policy preferences into account. Axelrod starts from a general hypothesis in chapter 1 of his book—which is about 'conflict of interest' in societies at large, not just in parliamentary settings—stating that "the greater the conflict of interest, the more likely is conflictful behavior" (p. 12). Conflict of interest is defined as "the state of incompatibility of the goals of two or more actors" (p. 10). In chapter 8, Axelrod applies this general hypothesis to the formation of multiparty coalitions in parliamentary democracies, deriving from it two predictions: (1) "The less conflict there is in a coalition, the more likely the coalition will form," and (2) "the less conflict of interest there is in a coalition, the more likely the coalition will have long duration if formed" (p. 167).

Axelrod conceives of parties as occupying a one-dimensional, ordinal policy space in which they are ranked from left to right (p. 168). The term 'ordinal' here means that the distance between each pair of adjacent parties is assumed to have no significance (ibid.). Furthermore, parties are assumed to have "single-peaked preferences", meaning their most preferred outcome is a single point in the policy space, and they prefer outcomes located closer to their preferred one in space to those located further away (ibid.). As it is impossible to measure the distance between parties in an ordinal policy space, the ideological dispersion within coalitions cannot be directly quantified (p. 169). Axelrod therefore limits his predictions to a set of qualitative properties. First, he predicts the formation of *connected* coalitions, in which all the constituent parties are adjacent to one another in the policy space (ibid.). In such coalitions, the ideological dispersion is minimal, and, by implication, the conflict of interest as well (ibid.). Aside from being connected, the predicted coalitions also have a minimal spread, meaning that a coalition of adjacent parties ABC, where A is the leftmost party and C the rightmost, is preferred over one consisting of adjacent parties ABCD, as the distance between A and D is per definition greater than that between A and C (p. 170). Hence, aside from connectedness, the size of a coalition matters, and this size should be minimal (ibid.). Of course, there is a lower limit on

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[6] Leiserson does not propose a term for his predicted type of coalition, so I resolved to call it the *minimal member coalition*, which is that W^m with the fewest number of members.

coalition size, because government coalitions must also be ‘winning’, that is, able to pass a vote of investiture, in systems where this procedure exists, or survive a vote of confidence, which usually amounts to having a simple majority of the seats in the lower house (ibid.). These three characteristics—connected, minimal and winning—together constitute the MCW, or the *minimal connected winning coalition* (ibid.).

Lijphart (1984) points out that minimal connected winning coalitions are not necessarily minimal winning coalitions in the original sense of the word, and can even be larger than the latter. In a 100-seat legislature with parties A, B, C, D and E, where A occupies 8 seats, B 21 seats, C 26 seats, D 12 seats and E 33 seats, and where A is the leftmost party and E the rightmost party, the combinations ABC (55 seats), ADE (53 seats), BCD (59 seats), BE (54 seats) and CE (59 seats) constitute minimal winning coalitions in the classical sense (p. 48). However, only the minimum winning coalitions ABC and BCD are minimal connected winning coalitions (ibid.). On the other hand, the coalition CDE, whose seat total equals 71, is not a minimal winning coalition in the numerical sense, as D can be removed without the coalition losing its majority. However, no party can be taken away from CDE without either lowering its seat share to under 50 (when removing C or E) or disconnecting the coalition ideologically (when removing D), and therefore it is a minimal connected winning coalition (ibid.). Volden & Carrubba (2004) derive from closed minimal range theory the prediction that whenever there are parties which are ideologically positioned in between sets of other parties that are otherwise able form minimum winning coalitions, oversized coalitions can be expected to form (p. 523).

Like Axelrod, de Swaan (1973) starts out from the assumption of an ordinal policy scale along which parties can be ranked from left to right (p. 88). The difference is that, rather than delimiting a predicted coalition type directly, as Axelrod does, de Swaan takes the perspective of the individual party. The ‘central behavioral assumption’ of his ‘policy distance theory’ states that “an actor strives to bring about a winning coalition in which he is included and in which he expects to adopt a policy that is as close as possible, on a scale of policies, to his own most preferred policy” (p. 88). A few observations are due here. In the first place, the fact that actors prefer to be members of winning coalitions does not follow from any axioms—it is explicitly assumed (p. 90). In zero-sum games, where the winners take from the losers, parties behaving rationally necessarily prefer to be part of the winning coalition to being a member of its losing complement, because only then they get a positive payoff. However, the game De Swaan describes is not zero-sum at all: there is no fixed quantity of a resource to be distributed, and moving the policy towards some participants and away from some others does not increase the payoff of the former with a quantity equal to the reduction of the payoff of the latter (pp. 101-102). For instance, coalition can simultaneously exclude a given actor while still adopting a policy position close to the preferences of that actor. In the absence of side payments or other benefits parties can obtain from being a coalition member, the only thing that matters to parties is the policy coalitions adopt (p. 101). To simplify things, then, he assumes that actors always prefer being a member of a winning coalition to being a member of a losing coalition. Second, since De Swaan uses an ordinal policy space, he runs into the same problem as Axelrod: ideological distance cannot be directly measured, and a coalition's adopted policy cannot be calculated based on the policy positions and weights of its constituent parties (p. 93).

De Swaan minimizes this problem, however, using an ingenious solution revolving around the 'pivotal actor'. Every possible coalition has a pivotal actor, that is, a party for which "the absolute difference between the combined votes (weights) of members [i.e., parties] on his right and of members on his left is not greater than his own weight", such that that this party is able to "swing the vote" in either direction (p. 89). For each coalition, then, the 'excess' of votes towards either side can be calculated by subtracting the combined weight of the members to the right of the pivotal actor from the combined weight of the members to the left of that actor, such that positive excesses indicate a bias toward the left and negative excesses a bias toward the right, and where the greater the excess, the more to the left a coalition's adopted policy lies (pp. 89-90). It then follows that "an actor's most preferred policy is closer to the expected policy of a coalition of which he is pivotal than to the expected policy of a coalition in which he is not pivotal" (p. 89). Furthermore, "the smaller the absolute excess of a coalition, the smaller the distance between the coalition's expected policy and the most preferred policy of the pivotal actor" (p. 90). Combining these insights with the central behavioral assumption, it follows that parties seek to enter winning coalitions in which they are the pivotal actor and of which the absolute excess is as small as possible. This enables one to rank all coalitions of which an actor is pivotal from most preferred to least preferred by that actor (p. 90).

But even some coalitions for which an actor is not pivotal can be ranked for that actor, albeit only those of which the pivotal actors lie on one side of said actor on the policy scale (p. 97). Then, a coalition with a pivotal actor closer to the actor from whose perspective we are comparing the preferability of coalitions—I will call him or her the point-of-view or POV actor[7]—is preferred by that actor to a coalition with a pivotal actor lying further away. However, when the pivotal actors of a pair of coalitions to be compared lie on either side of the POV actor, the distances between the POV actor of interest and those pivotal actors cannot be compared due to the ordinal nature of the policy scale (ibid.). In this case, the preferences of the POV actor regarding those two coalitions are left 'indeterminate' (p. 98).

For every actor in the game, then, all possible coalitions can be ranked from most preferred to least preferred. The rank-orders for each actor can then be combined into a 'preference matrix',

[7] De Swaan does not use this term; I introduced it in order to facilitate the explanation of the model.

where each column represents an actor, each row represents a possible coalition, and each cell contains the rank number of a given coalition for a given actor (p. 107). Recall that some coalitions cannot be compared because of the indeterminacy of preferences mentioned above, and coalitions for which this is the case get a few possible rank numbers, for instance '2 or 3' or '1, 2 or 3' (ibid.).

Based on this, De Swaan does not predict one specific 'most preferred' imputation like Riker and Leiserson did, but rather derives a solution similar in kind to that of N-M: "a set of imputations that are in some kind of equilibrium" (p. 35). However, De Swaan is dissatisfied with the N-M solution concept, which, as was pointed out before, consists of an infinity of imputations (ibid.). He is more enthusiastic about another solution concept, known as the 'core'[8], which differs from that of N-M in an important respect: it consists of the set of imputations that are undominated (ibid.). Whereas the N-M solution allows its imputations to be dominated by imputations that are not part of the solution, the core does not. The reason N-M use their 'weaker' solution, De Swaan points out, is because those authors are mostly interested in zero-sum games, and the core is empty in such games[9], meaning there are no undominated imputations (p. 36). De Swaan's game, on the other hand, is non-constant-sum, and the core exists in such games (p. 37). Therefore, De Swaan is able to use the core as a solution concept.

By the elimination of dominated imputations from the preference matrix, one arrives at a set of undominated imputations. In a specific example, where the parties *a*, *b*, *c* and *d* are ranked from left to right in that order and hold respectively 32, 23, 30 and 15 seats, two certainly undominated coalitions emerge: *a* & *b* & *c* and *a* & *b* & *c* & *d* (pp. 107-108). Then there are two other coalitions, *a* & *b* and *b* & *c* & *d*, either of which may be undominated or 'weakly' dominate one another as a result of the indeterminacy of preferences (p. 108). Nevertheless, these two are still part of the core, because they cannot be demonstrated to be unstable (ibid.). Note that *a* & *b* & *c* & *d* is a grand coalition consisting of all parties. It becomes clear now that De Swaan's theory does not predict any kind of 'minimal' coalition in terms of size, weight, number of members or even ideological range (ibid.). As the policy adopted by a coalition is the only thing that matters for the participants, size is not relevant, only the distance between the policy adopted and their own preferred policy (ibid.). From the perspective of a pivotal actor, then, the only matter of import is the balance between the weights on either of his sides, not

[8] De Swaan borrows this concept from Gillies (1959).
[9] The core is empty in constant-sum games more generally, as long as they are essential (De Swaan, 1973, p. 36). That is because in constant-sum games, the payoffs to the participants always sum to the same value (p. 37). Therefore, it is not possible to create an imputation under which all actors are collectively better off than under any other, because a gain for one of the participants automatically implies a loss for another (ibid.). Inessential games, on the other hand, have a single imputation as a solution, that is, no coalition but each participant playing for himself (ibid.).

their absolute sizes. If adding additional parties over and above the required minimum provides a pivotal actor with the best balance, she will gladly accept it (ibid.).

In order to improve the realism of his theory and assign a greater role to distance rather than just weight, De Swaan adds two independent modifications to be tested alongside the main model. The first modification, named the 1,*n* permissibility condition, prohibits coalitions from including ‘extreme’ actors, meaning the leftmost party 1 and the rightmost party *n*, if those actors are not better off in that coalition than they would be in the coalition of all members, the grand coalition *N* (p. 115). The grand coalition itself, then, is excluded by definition, because members of *N* cannot be better off in *N*, because they are already members of *N* (ibid.). In the above example, this would lead to the elimination of three coalitions from the preference matrix: *a* & *b* & *c* & *d*, the grand coalition, *a* & *c* & *d* (*a* is not better off than in *a* & *b* & *c* & *d*), and *a* & *b* & *d* (*d* is not better off than in *a* & *b* & *c* & *d*) (p. 116). This results in the exclusion of *a* & *b* & *c* & *d* from the core, and the inclusion of *b* & *c*, which used to be dominated by the now impermissible *a*, *b* & *d* (pp. 116-117).

The second modification, the *closed coalition condition*, eliminates all *open coalitions*, which are those that are not connected in the sense of Axelrod (p. 117). This condition can either be added to the existing theory, eliminating open coalitions from the set of permissible coalitions from which the core is computed, or it can be used as an independent “closed coalition proposition”, which predicts that closed winning coalitions will form (p. 118). The difference between this proposition and Axelrod’s minimal connected winning coalition appears to be the fact that the closed coalition proposition does not require coalitions to be minimal (ibid.). Furthermore, this proposition is independent of game theory and even rational choice theory, instead being an “independent generalization on the behavior of political actors” (ibid.).

Variables Affecting Coalition Size

The theories discussed up to this point predicted (sets of) coalitions to form on the basis of a number of premises or axioms. Later theories modeled the impact of certain independent variables on the formation of coalitions. Those theories—or at least those making predictions about coalition size—are discussed in this section.

Information and the Willingness to Bargain

As mentioned in the section on the minimal winning coalition, Riker argues that under the condition of perfect information, coalitions are always minimum winning (p. 32). However, shortly after laying out the aforementioned argument, Riker relaxes the perfect information condition and formulates the size principle in the following way: “In social situations similar to n-person, zero-sum games with side-payments, participants create coalitions just as large as they believe will ensure winning and no larger" (p. 47). In a subsequent chapter, he elaborates on the role of information in coalition building. In the uncertain real world, coalition builders are often forced "to aim at a subjectively estimated minimum winning coalition rather than at an actual minimum" (p. 77). Riker distinguishes between two types of information certainty: *complete information*, when one participant knows the precise weight of another participant, and *perfect information*, when a participant knows what previous moves another participant has made (p. 78). If all participants have information about all others, the adverb *systematically* is added, and so *systematically complete information* means that all participants are aware of one another's weights and *systematically perfect information* means all actors know about one another's previous moves (ibid.). In legislatures, each member has one vote and so information is effectively systematically complete (p. 80). On the other hand, information in such contexts is not (systematically) perfect because the participants often do not know about one another's moves (ibid.).

When information is not systematically complete and perfect in such "natural decision-making bodies", Riker argues, "coalition makers tend to aim at forming coalitions larger than the minimum winning size" (p. 81). More specifically, the greater the degree of incompleteness and imperfection of information, the larger coalitions will be, and conversely, as information certainty approaches completeness and perfection, coalitions approach minimum winning size (pp. 88-89). This Riker calls the 'information effect' (p. 89).

Dodd (1976) applied Riker's model to the formation of coalition governments in parliamentary systems (p. 35). In doing so, he adapts the model to incorporate an additional variable: actors' a priori willingness to bargain (p. 34). In general, his assumptions are the following. First, parties seek governmental power, that is, they desire to enter the cabinet and maximize their number of portfolios (pp. 35-36). Second, in return for their payoff in portfolios, parties bring their own resource to the table, that is, the legislative votes they possess (p. 36). Third, parties employ a basic criterion in coalition negotiations: they seek to enter a minimum winning coalition (p. 37). Coalitions that are not minimum winning are either non-winning or oversized (ibid.). Parties seek to avoid the former because it does not guarantee victory, while they seek to avoid the latter because it 'wastes' payoffs, that is, it does not maximize ministries for the individual party (ibid.). So far, these assumptions do not differ from those of Riker.

The fourth and final assumption, however, constitutes a partial departure from Riker, and states that parties are constrained in their negotiations by the bargaining conditions that prevail in parliament (p. 38). These bargaining conditions have two dimensions: information certainty and the a priori willingness of parties to bargain (p. 40). Following Riker, Dodd defines information certainty in terms of two dimensions: complete information as to weights, or the reliable number of legislative votes parties command[10], and perfect information as to prior moves, that is, previous offers, counteroffers and bargains made among the parties in coalition negotiations (p. 40, citing Riker pp. 77-89). A general willingness to bargain means that all parties are willing to consider entering into a coalition with any other party (p. 40). When the general willingness to bargain is high, majority cabinets should form (p. 41). By inversion, a low willingness to bargain produces a tendency toward minority cabinets (ibid.).

The willingness to bargain interacts with information certainty in the following way: in parliaments with low bargaining constraints, if parties lack information about one another's weights and previous moves, they cannot accurately gauge the reliability of one another as coalition partners (p. 44). To compensate for this uncertainty, parties form oversized coalitions (ibid.). The greater the uncertainty, the larger the surplus size (pp. 44-45). On the other hand, in parliaments with high bargaining constraints, imperfect information leads to coalitions that are smaller than would have been possible with perfect information (p. 46). Given that winning coalitions are not possible in this situation, parties seek to at least enter the largest possible coalition, as this reduces the chances of a larger rival coalition forming (p. 45). However, given imperfect information regarding their potential partners' previous moves, parties run the risk of being drawn into coalitions with undesirable partners by otherwise desirable partners (ibid.). Caution will then lead them to opt for smaller coalitions rather than risk ending up in a coalition with undesirable partners. At the same time, uncertainty as to weights complicates calculations and negotiations as parties cannot accurately estimate the ideal coalition size (ibid.). Negotiations will therefore take longer, and, time being a scarce resource, parties may have to settle for non-ideal, smaller coalitions rather than enter into a coalition with partners undesirable to them for a priori reasons (p. 46). Note that Dodd is the first of the discussed authors who explicitly relaxes the majority requirement for winning coalitions, acknowledging that a minority coalition can survive as long as no alternative majority coalition emerges to challenge it. This insight would be important for later theories on minority governments.

In general then, irrespective of the general willingness to bargain, minimum winning status is approached as information certainty is approached (ibid.). In parliaments with low bargaining constraints, uncertainty leads to coalitions that are larger than ideal, whereas in parliaments with high constraints, uncertainty leads to coalitions that are smaller than ideal. As such, the unifying logic of this argument is that, given the existing constraints on the willingness to bargain, information uncertainty leads parties to settle for outcomes that are less than ideal.

[10] While the number of seats a party formally controls can be readily consulted, the degree of its internal voting discipline affects the number of votes a party can reliably count on (pp. 40, 44).

Institutions

In the theories discussed so far, little attention has been paid to institutions and their rules. Rather, coalition formation was explained purely on the basis of abstract principles and motivations. When considering coalition formation within a specific institutional context, however, which is what we are doing in this section, then the rules of that institutional context are among the most basic determinants of coalition formation. Two of such institutional structures or rules are discussed here: bicameralism and constitutional amendment rules. These considerations, however, are compatible with game theoretical models and do not necessarily alter the basic motivations of the actors as assumed by the different models.

Lijphart (1984) pointed out that in systems characterized by what he calls 'strong bicameralism', coalitions often have to be oversized in order to secure a majority in both houses[11] (p. 104). The importance, or even necessity, of securing a majority in the upper house in systems characterized by strong bicameralism can be readily understood after considering the definition of that term. Parliaments must meet two criteria to be considered strongly bicameral: they must be *symmetrical*, meaning that the formal powers of their two houses are equal, and *incongruent*, meaning that their composition differs as a result of different selection methods (pp. 96-99). For instance, in some countries the members of the lower house are elected by proportional representation, while the members of the upper house are elected in a non-proportional way, for instance, districts of different population sizes may all get to elect a fixed number of representatives independently of population size (p. 98). This often gives the upper house a rural bias (ibid.). In other cases, the members of the upper house may not be popularly elected at all, but rather, determined by inheritance, such as in the British House of Lords, giving that house a conservative bias (ibid.).

In parliamentary systems with strong bicameralism, the symmetry between the houses of parliament also means that the cabinet is dependent on the confidence of both (p. 101). Thus, governments require a majority in both houses to survive. As the houses of government in such systems are also incongruent (by definition of strong bicameralism), those majorities will differ in their partisan compositions. As such, reproducing the exact same minimal winning coalition in terms of partisan composition and relative size that exists in the lower house in the upper house may not suffice for a majority. Instead, a coalition must be built that simultaneously holds a majority in both houses. What then appears to be an oversized coalition in the lower house may in fact be a minimal winning coalition in the upper house. Hence, the insights of different theories can still apply, it is just that a provision must be made for strong bicameralism.

[11] Sjölin (1993) made a similar observation (p. 102).

But majority coalitions in parliamentary systems do not just exist to guarantee the confidence of the legislature, they also serve to facilitate lawmaking. When the majority threshold for the approval of law bills coincides with that of parliamentary confidence, those two aims are both fulfilled automatically by a single minimal winning coalition. However, when the majority threshold for lawmaking differs from the one for confidence, such as in the case of constitutional amendments, the minimal winning coalition for lawmaking (‘winning’ here meaning to pass a constitutional amendment) will be larger than the minimal winning coalition for confidence (‘winning’ here meaning to stay in power). Lawmaking then becomes the more stringent requirement and cabinets seeking to pass constitutional reforms or other types of legislation requiring a special majority will be at least as big as the threshold.

Policy Influence and Electoral Calculations

In the discussion on Dodd, some conditions under which coalitions are likely to be of minority size were already identified. Scholarship on minority governments in parliamentary systems gained another major impetus with the 1990 publication of Kaare Strøm’s seminal book *Minority Government and Majority Rule*. Strøm argued that minority governments in parliamentary systems were not the result of crisis and dysfunction, as was commonly believed at the time, but rather of the rational decisions of the involved actors (p. 16). To this end, he presented a rational choice theory that was innovative in its assumptions. These are: (1) there is a difference between executive coalitions (made up of parties with cabinet representation) and legislative coalitions (consisting of parties supporting the government in parliament, whether or not they occupy cabinet seats), (2) in addition to office and policy, parties seek to maximize their votes, as doing so improves their bargaining position, (3) government participation is not a necessary condition for policy influence, (4) there is a temporal component to party behavior in the sense that holding office yields short-term benefits while carrying electoral costs in the long run, and (5) this trade-off between office and electoral success affects decision-making in relation to cabinet formations (p. 69).

A number of predictions follow from these premises. By virtue of assumption 3, in systems where the opposition has considerable influence over policymaking, the relative benefit of holding office decreases and policy-seeking parties are more likely to forego it, and, by virtue of assumptions 2, 4 and 5, in systems where elections are highly decisive for government participation, vote-seeking parties are more likely to avoid office in the short run in order to get a better position at the bargaining table in the long run (pp. 69-70). This means that, in systems where the two aforementioned conditions hold, the ‘parliamentary bases’ of coalitions can be expected to be smaller, and minority governments to be more common (p. 70). By extension, we should then expect to see larger coalitions in polities where the opposition has little influence over policymaking and elections are indecisive. Strøm identifies a number of proxy variables for the two abovementioned dimensions. The strength, size and degree of specialization of committee systems are taken as indicators for the extent to which opposition parties can influence policymaking (p. 71). The identifiability of government alternatives, the volatility of elections, defined as the changes in vote shares occurring from election to election, the responsiveness of elections, defined as the correspondence between votes and seats, and the temporal proximity of government formations to elections are used as indicators for the decisiveness of elections (pp. 72-74).

One objection that can be made to Strøm's argument is that it is not necessary for non-cabinet parties to influence policy in order to benefit from it. For instance, De Swaan admitted that office-seeking motivations did not follow from his policy-based model, and implicitly suggested that parties can be in the opposition and still benefit from the policy adopted by the government. This insight is taken to its logical conclusion by the models that will be discussed next.

Proposal Credibility

As the theories of De Swaan, Strøm, and to a lesser extent, Axelrod demonstrate, when policy considerations are given a greater role, coalition size becomes relatively less important, as policy is not a resource to be divided among the participants, that is, games in which policy is the main aim of the participants are not zero-sum. Another theory that reinforces this idea is by Laver & Shepsle (1990). Following authors such as Schofield (1986) and McKelvey & Schofield (1987), Laver & Shepsle (1990) moved away from the one-dimensional policy scale of Axelrod and De Swaan to a multidimensional policy space. However, whereas Schofield and McKelvey & Schofield allowed policy outcomes at any point in the policy space, Laver & Shepsle made an innovation which limited the set of possible policy outcomes substantially, facilitating analysis (p. 875). Laver & Shepsle conceived of the cabinet as possessing a 'division of labor' in which individual ministers determine what happens within their assigned portfolios (p. 874). Furthermore, parties that are assigned portfolios are forced to implement policies in accord with their stated preferences (ibid.). Expecting parties to implement policies that are at odds with their stated positions would not be credible (ibid.). Hence, the analysis is limited to 'credible proposals', meaning those in which parties controlling portfolios implement policies that are in agreement with their own preferences (ibid.). The number of credible proposals is finite and can actually be quite small depending on the number of portfolios and parties (ibid.).

In order to represent credible proposals visually, Laver & Shepsle use the concept of the lattice, a roster in which the y-axis represents policy in one ministry and along the policy-dimension (e.g. defense policy) controlled by that ministry and the *x*-axis represents policy in another ministry and along another policy dimension (e.g. economic policy) (pp. 874-875). Of course, there can be any number of portfolios and corresponding policy dimensions, but for this specific example they assume the existence of two portfolios and three parties, *A*, *B* and *C* (ibid.). Each party then has an ideal point in the policy space, with an *x*-coordinate representing its position on dimension *x* a *y*-coordinate representing its position on dimension *y* (ibid.). A proposal, then, refers to a specific distribution of portfolios among the possible coalition partners. A proposal is represented by two letters: the first one for the party controlling the *x*-portfolio and the second one for the party controlling the *y*-portfolio (pp. 874-875). For instance, *AA* is a proposal in which party *A* controls both portfolios and *AB* is a proposal where party *A* controls portfolio *x* and party *B* controls portfolio *y* (p. 875). With three parties, there are nine possible portfolio distributions.

By virtue of the credibility condition, a party controlling a ministry, let's say party *A* controlling ministry x, cannot deviate from its position on the corresponding policy dimension; it is stuck to the vertical line passing through its position on the x-axis. Hence, each distribution of portfolios corresponds to a single policy outcome, a fixed point in the policy space. For instance, the position of the credible proposal *AB* in the policy space has as its *x*-coordinate *A*'s position on the corresponding policy dimension *x* and as its *y*-coordinate *B*'s position on the corresponding policy dimension *y*. Fixing policy outcomes to portfolio distributions then means that the number of credible proposals and their corresponding policy outcomes is equal to the number of possible portfolio distributions, that is, nine (p. 876). This is clearly much more manageable than allowing policy outcomes to occupy any of the infinity of points that exist in a two-dimensional space.

[insert figure: lattice with credible proposals but without indifference curves. Make reference to figure in text.]

The rest of the article then revolves around determining which proposals defeat or dominate which other proposals, in a similar fashion to N-M (1953) and De Swaan (1973). Laver & Shepsle identify three kinds of domination relationships. The first is when one proposal *x'* is preferred by a legislative majority to another proposal *x*, where the set of credible proposals that *win* against *x* in the legislature is called the *credible win set of x* or *W*(*x*) (p. 876). The second is when *x'* is preferred by its participants (the parties holding portfolios) to *x*, where the set of proposals that are considered an *improvement* to *x* by its participants is called the *improvement set of x* or *I*(*x*) (p. 877). Some proposals may win against others in the legislature but are not preferred by all of their participants. Let us say party *B* is a participant of both *x* and *x'*, and *x'* wins against *x* in the legislature, but *B* does not prefer *x'* to *x*. Then $x' \in W(x)$ but $x' \notin I(x)$. If *x'* wins against *x* *and* is preferred to *x* by all of its participants, *x'* is said to *contend* against *x* and the set of proposals that do so is called the *contender set of x*, $C(x) = W(x) \cap I(x)$ (ibid.).

Whether an actor prefers one proposal over another depends on the relative distance of either one to its ideal point. If *x'* lies closer to a party's ideal point than *x*, then that party prefers *x'* to *x*. Similarly, a legislative majority prefers *x'* to *x* when the ideal points of a set of parties that together hold a legislative majority lie closer to *x'* than to *x*, whether or not they are participants of either proposal. This insight allows one to draw *indifference curves* for each party, which in turn makes it possible to compare alternative proposals to the status quo proposal x^0 (p. 877). In order to do this, one must draw indifference curves for each participant that intersect with x^0. The credible proposals lying within the union of the intersections of the interiors of the indifference curves of a (weighted) majority of the parties then constitute the credible win set of x^0 (ibid.). If a winning proposal also lies closer to the ideal points of all of its participants (again: the parties holding portfolios in it) than x^0, it contends against x^0.

[insert figure with indifference curves (figure 1 in the paper)]

A status quo facing contenders is said to be *vulnerable*, while a status quo that does not face any contenders is said to be *invulnerable* (ibid.). The set of invulnerable or undominated proposals then constitutes the *restricted portfolio core* (pp. 877-878). Recall from the discussion on De Swaan that the core is the set of undominated imputations. Laver & Shepsle borrow the term 'restricted portfolio core' from Austen-Smith & Banks (1990)[12], who demonstrated that the restricted portfolio core always exists with three parties but not necessarily with four (cited by Laver & Shepsle, 1990, p. 878).

Laver & Shepsle derive results for a number of different scenarios. In the first place, an invulnerable status quo always remains intact—meaning that the incumbent government stays in office—even if there are proposals that win against it in the legislature, but that are not preferred to it by their participants (pp. 878-879). When the status quo is vulnerable only to invulnerable contenders, one of two things can happen. In case of a single invulnerable contender, the status quo is replaced by that contender, but in case of two invulnerable contenders, a so-called *Mexican standoff* ensues, in which the result is indeterminate: either one of the invulnerable contenders replaces the status quo, or the status quo remains in force (pp. 880-881). Finally, if the status quo is vulnerable only to vulnerable proposals, there are again two sub-scenarios. First, if there are no invulnerable contenders, there are *contender cycles* in which every proposal loses and is preferred by its participants to some other proposal (p. 881). This situation seems to be equivalent to the cyclical domination or intransitivity of N-M's simple games. As already indicated in the discussion of De Swaan, the core is empty in such situations, which Laver & Shepsle also note (ibid.). In a real-world context, this situation translates to chaos (ibid.).

[12] Like Laver & Shepsle, Austen-Smith & Banks (1990) use a model based on portfolio allocations. They do however not restrict the analysis to what Laver & Shepsle call credible proposals, and so their results are less determinate than those of Laver & Shepsle.

Institutional procedures and behavioral norms may partly resolve this chaos, but "the life of governments may be short and elections frequent" (ibid.). By far the most complex is the second sub-scenario of a status quo vulnerable only to vulnerable contenders: here, there is an invulnerable contender x^* that does not contend against x^0 (p. 881)! This can happen for instance when x^0 is vulnerable to some proposal x' and x' is in turn vulnerable to x^*, but x^0 is not directly vulnerable to x^* (ibid.). The result here is again largely indeterminate. If x^* wins against x^0 in the legislature, meaning that $x' \in W(x^0)$ and $x' \notin I(x^0)$ (recall that x^* does not contend against x^0), there may or may not be ways to reach it that cannot be blocked (pp. 881-882). On the other hand, if x^* loses against x^0 in the legislature, meaning $x' \notin W(x^0)$, x^* is *strategically irrelevant* and the situation is equivalent to that of the contender cycles mentioned above (p. 882).

However, the main reason that Laver & Shepsle's (1990) model was included in this chapter is the fact that it has something to say about coalition size. As was already mentioned before, for a proposal x' to contend against the status quo x^0, it must be at least preferred by a legislative majority to x^0, but this does not mean that the participants (portfolio holders) of x' themselves must constitute a legislative majority (p. 885). Put differently, it is possible for a group of parties constituting a legislative majority to support a government proposal in which not all of its members hold portfolios. This insight is very similar to that of Strøm's distinction between executive coalitions and their parliamentary bases, whereby a minority cabinet may still enjoy majority legislative support[13]. When a single invulnerable proposal contends against the status quo, and that proposal corresponds to a minority government, nothing can contend against that minority government and so it will be realized (Laver & Shepsle, p. 885). Conversely, there are situations in which surplus parties may be invited to a government and put in charge of key portfolios in order to make the proposal credible (ibid.). Parties are then really only redundant if their removal affects neither the vulnerability nor the credibility of the proposal (ibid.). In an endnote, Laver & Shepsle point out that this finding does not contradict but rather complements Riker's minimal winning thesis (p. 890). After all, they argue, Riker's thesis can be reinterpreted as meaning that all members of a coalition must be both necessary and sufficient for it to win (ibid.). In an "institutionally more elaborate" context, the meaning of necessity and sufficiency can be different from that in the simpler context on which Riker wrote, and so some members of a minimal winning coalition in Laver & Shepsle's universe may be redundant in terms of their legislative basis, but necessary for the credibility of the proposal (ibid.).

[13] Strøm's book was published in the same year as Laver & Shepsle's paper, but just a few months earlier (April versus September 1990).

The Party System

The next few theories attempt to link the characteristics of party systems to the kinds of coalitions most likely to be observed. For instance, Schofield (1987) links the three different European party systems identified by Daalder (1971) to specific coalition outcomes. The main argument of this paper is most clearly summarized by Schofield (1993). In *unipolar systems*, such as the Scandinavian countries Norway, Sweden, Denmark and Iceland, there is one large party and three or four smaller parties (Schofield, 1993, p. 2). Here, minority governments consisting of the large party, sometimes supported by one of the other parties, are common (ibid.). In *bipolar systems*, such as Germany and Austria, there are two large parties and one or two smaller ones, and government coalitions tend to be both minimal winning and minimal connected winning (ibid.). *Multipolar systems*, such as Belgium, Luxemburg and Iceland (Iceland appears twice), are characterized by the existence of two larger and at least two smaller parties, and coalitions tend to be minimal winning, but not necessarily minimal connected (ibid.). Finally, in *fragmented systems*, such as the Netherlands, Finland and Italy, there are two or three large and four or five small parties, and coalitions, usually consisting of one of the bigger and two or more of the smaller parties, tend to be oversized and short-lived (ibid.).

At first glance, it seems as if these arguments can be summarized as 'the greater the fragmentation of a party system, the larger the coalitions'. Indeed, whereas bipolar and multipolar systems are characterized by a predominance of minimal winning coalitions, fragmented systems are associated with a higher incidence of oversized coalitions. However, looking at Schofield's (1987) party system data, it becomes clear that the unipolar systems generally display higher fragmentation than the bipolar systems and even some of the multipolar systems (pp. 577-580), and still, they are characterized by a high incidence of minority governments. In those systems, the dominance of one party more or less guarantees that party of government membership, even allowing it to form minority governments. Such a party, which, due to its more or less central position in the policy space and size can guarantee itself membership of every government coalition, is called a *core party* by Schofield, alluding to the game-theoretical solution concept of the core (Schofield, 1993, pp. 5-6). Core parties are "able to influence coalition bargaining in order to control the policy position of the government" (p. 8). As such, if policy is assumed to be the main motivation of parties, core parties are indifferent to the specific coalition that forms, and their ability to control policy means they can fulfill their policy objectives without the help of other parties, forming a minority government (ibid.).

Laver & Shepsle (1996) make a similar argument. On the basis of a model more or less identical to that of Laver & Shepsle (1990), which was discussed in the previous section, they define a *strong party* as one that participates in every cabinet that is majority-preferred to the one in which it holds all portfolios (pp. 69-70). There are two ways in which this can be the case. Either the single-party government containing that party has an empty winset, meaning no other cabinet is majority-preferred to it, or, if that one-party cabinet has a non-empty winset, said party participates in every cabinet that is majority-preferred to the one-party cabinet (p. 70). In the first situation, the party in question is said to be *very strong*: it can impose its will on the remainder of the parties and form a minority, single-party cabinet (pp. 70-71). This situation seems to be equivalent to the invulnerable status quo of Laver & Shepsle (1990). In the second case, the party under consideration is said to be *merely strong*: while it cannot directly impose a one-party cabinet, it can veto all cabinets in the winset of that one-party cabinet, because it is a participant of those cabinets (Laver & Shepsle, 1996, p. 70).

Strong parties have a number of interesting properties. First, there can either be one strong party[14] or no strong party at all (p. 71). Second, very strong parties lie at the generalized median[15] point of the parties in the policy space[16] (p. 70). Third, while a single-party government formed by a very strong party is always in *equilibrium* (p. 71), meaning it will stay in place as no political actor with the ability to bring it down has the incentive to do so (p. 61), this is not necessarily the case for single-party governments formed by merely strong parties. While a merely strong party participates in every equilibrium cabinet, it “cannot necessarily impose its ideal point as the outcome of the government formation process” (pp. 73-74). Laver & Shepsle provide an example of a situation in which the most a merely strong party B can do is keep vetoing the government, say, AB, that is in the winset of its ideal point, BB, while A, which prefers AB to BB, in turn keeps vetoing BB (p. 75). This constitutes an unresolvable standoff (ibid.).

The key take-home message from Laver & Shepsle’s (1996) theory for our purposes is that strong parties, like Schofield’s (1993) core parties are “central in some sense” (Laver & Shepsle, 1996, p. 78), and that this property allows them to form governments alone under certain circumstances. However, the most elaborate and rigorous defense of this idea, that central parties have some kind of bargaining advantage that allows them to form small or even minority governments, comes from Crombez (1996).

Crombez (1996) models the coalition formation process as a multistage game. He assumes a parliament with three parties of varying sizes, which are numbered 1 to 3, with party 1 being the

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[14] This logically follows from the definition of the strong party. If there were two strong parties, S_1 and S_2, then the ideal point of one of them would have to be in the winset of the other (p. 71). However, if, for instance, S_1 is in the winset of S_2, there must be a proposal in the winset of S_2 in which S_1 does not participate (ibid.). This violates the definition of the strong party (ibid.).
[15] The generalized or geometric median of a set of points in a multidimensional space is that point from which the summed distances to the other points in the set is minimal **(ref.)**.
[16] Laver & Shepsle base this proposition, that an ideal point with an empty winset is at the generalized median, on Kadane’s Theorem (1972) (p. 70).

largest and party 3 the smallest, where none holds an absolute majority of the seats in the legislature (p. 5). Parties are office-seeking and policy-seeking (ibid.), meaning that holding office is not just seen as a means to influence policy but is valued in its own right. Like Schofield (1987, 1993) and Laver & Shepsle (1990, 1996), Crombez conceives of parties as occupying a position in a multidimensional policy space (p. 5). In the first stage of the game, party 1 is selected to be the formateur (p. 6). This party is then allowed to propose a government, which may be minority, minimum winning or surplus majority (ibid.). If the proposal is accepted, the game enters the third stage, in which the coalition partner(s) can decide whether or not they want to defect and trigger a confidence vote, or continue to support the coalition (p. 7). If the coalition is oversized and includes all three parties, and one party withdraws support, the formateur, in stage 4, gets the chance to reshuffle cabinet portfolios and adjust policy before the confidence vote in an attempt to retain the support of the remaining party and continue in office (ibid.). If the coalition is minimal winning and includes only two parties, however, the defection of the junior partner automatically triggers the collapse of the government (p. 9). Note that the formateur is not allowed to negotiate a coalition agreement with the opposition party (pp. 7-8). In stage 5, the legislature votes on the confidence motion, needing an absolute majority to pass (p. 8). If the motion fails, the government stays on until the end of the legislative term (ibid.). If the motion passes, the government falls and a new formateur is selected in stage 6, with the probability for a party *i* to be selected as the formateur being equal to its seat share π_i (ibid.). This formateur can then make a proposal in stage 7, which is voted on in stage 8 (ibid.). If the proposal is accepted, the newly formed government stays on for the remainder of the legislative term (ibid.). If the proposal is rejected, a caretaker government takes office, yielding zero utility for all parties (ibid.).

If the initial proposal is rejected by at least one prospective coalition partner in the second stage, a new formateur is selected, with a party's selection probability being equal to the average of its selection probabilities in the first stage and the sixth stage (p. 8). For parties 2 and 3, who have zero chance of being selected in the first stage, this comes down to their seat shares divided by two, respectively $\frac{\pi_2}{2}$ and $\frac{\pi_3}{2}$ (p. 7, figure 2). For party 1 however, it is equal to one—its stage 1 selection probability—plus its seat share, divided by two: $\frac{1+\pi_1}{2}$ (ibid.). Party 1 is thus highly likely at this stage to be selected again (ibid.).

The main prediction that follows from this model is that the size and centrality of the largest party, party 1, negatively impacts coalition size (p. 9). In order to understand why this is the case, two facts must be considered. First, the larger a party is, the greater the chance it will be selected as the formateur at any stage, and the greater its utility after the collapse of a government which it did not propose (ibid.). Second, the more central a party is, the closer government policy will be to its own position, whether it is included in the government or not, and the greater its utility after the collapse of a government it did not propose (ibid.). By extension, as party 1 increases in size and becomes more central, the other parties become smaller and less central, decreasing their utility after the collapse of party 1's government (ibid.).

Let us now consider two scenarios. The first scenario is one in which party 1 proposes a minimum winning coalition with party i. Let v_i^f denote the utility of party *i* after rejecting party 1's proposal in stage 2, leading to the failure (*f*) of the formation attempt (p. 9). Let v_i^c denote the utility of party *i* after defecting from and triggering the collapse (c) of the government in stage 3 (ibid.). Because party i's selection probability is higher after the collapse of the government than after the failure of the formation attempt, v_i^c is greater than v_i^f (ibid.). Therefore, if party 1 wants to keep party *i* in the government, it must offer it a utility equal to v_i^c in the form of policy concessions and portfolios (ibid.). Because the smallest party, party 3, has a smaller selection probability than party 2, v_3^c is smaller than v_2^c, and therefore it is cheaper for party 1 to invite party 3 to the coalition than party 2 (ibid.). Now, there is a point where party 3 is so small and distant from the center (corresponding to the fact that party 1 is large and central) that only getting a government policy between its own ideal point and that of party 1 and no portfolios already yields it more utility than collapsing the government and get a chance to become the formateur (p. 21). The alternatives for party 3 are all worse: in a one-party government led by party 1, a coalition between party 1 and party 2 or even a coalition between party 2 and party 3, the distance between the policy outcome and party 3's ideal point is always greater than in a coalition between parties 1 and 3. In this case, party 1 does not have to offer party 3 any portfolios and forms a minority government supported by party 3 (pp. 9-10).

On the other hand, when party 1 is relatively small and peripheral, the above will not work because party 3 will be relatively large and obtain substantial utility from collapsing the government as it is more likely to be selected as a formateur in subsequent formation rounds. In order to build a lasting government, then, party 1 invites party 2 *and* party 3 to an oversized coalition (p. 10). This creates a prisoner's dilemma: if either junior partner decides to jump ship in stage 3, party 1 reshuffles the cabinet, adjusts government policy and continues in office in a coalition with the remaining partner (ibid.). The defected party now has a negative utility v_i^r (r stands for reshuffle): it lost its cabinet portfolios and now has a less favorable government policy (ibid.). On the other hand, the remaining partner is now better off than before, with a more favorable government policy and portfolio distribution (ibid.). In equilibrium, then, both junior partners stay in the coalition (ibid.). Either prospective coalition partner does still obtain some utility from rejecting party 1's proposal in the second stage, however, and so party 1, not wanting to risk losing its formateur status, offers both of them a utility equal to v_i^f (ibid.).

Crombez follows Strøm and Laver & Shepsle (1990, 1996) in the sense that he acknowledges that policy is a payoff that parties can obtain independently of them holding office. As a result, Crombez, like Strøm, does not see minority governments as a sign of malaise or crisis. Instead, he sees minority governments essentially as a sign of formateur party strength (p. 2). The existence of a minority government implies that the formateur did not need to share portfolios with its coalition partners, who are paid off in the form of policy, by virtue of its strong bargaining position (ibid.).

The Status Quo

Whereas Crombez focuses on the impact of party size and centrality on coalition size, Baron & Diermeier (2001) explore the effect of the status quo in their model, which is based on many of the same assumptions as Crombez's: there is a parliament with three parties which are motivated by policy as well as office, office benefits can be exchanged for policy concessions, and the coalition formation process has multiple stages, three to be precise (pp. 935-939). There is an electoral stage in which the parties' seat shares are determined, a government formation stage and a legislative stage in which a policy is agreed upon (p. 939). After the electoral stage, a formateur is selected by chance, with the probability of a party being selected equal to its seat share (ibid.). A party holding an absolute majority of the seats is automatically selected as the formateur (ibid.). The formateur selects a so-called 'proto-coalition', a set of prospective coalition members, which then negotiate over policy and the distribution of office-holding benefits (p. 940). Here, the formateur can make a take-it-or-leave-it offer[17], meaning that the other parties are not allowed to make counteroffers (p. 941). If the offer is accepted, a government is formed with a policy and distribution of office benefits that are characterized in a subsequent section. If this offer is turned down, a caretaker government is installed, which results in the continuation of the status quo while no office benefits are distributed among the parties (p. 940). This is one of the main differences with Crombez: there is only one chance to form a government and only one party gets to make a proposal.

A second major difference arises from the fact that Baron & Diermeier incorporate voter preferences into their model, which Crombez does not. Voters have preferences over policy and are aware of the policy positions of each party (p. 941). They vote prospectively based on their

[17] Baron & Diermeier also explore a scenario with multiple rounds of formation attempts (pp. 950-951). This does not change the basic intuition of their results, while it does increase the reservation value (the minimum amount a seller is willing to accept) of the non-formateur parties, which can lead the formateur to offer office holding benefits to prospective coalition partners (p. 951).

knowledge about the policies that all possible single party and coalition governments would implement (ibid.). The resulting electoral equilibrium causes coalition governments to always adopt policies that lie at the centroid of the coalition partners' ideal points (p. 953). A third difference is that, while Crombez does not make any a priori assumptions about the relative positions of the parties in the policy space, Baron & Diermeier set the parties' ideal points equidistant from one another, such that they form an equilateral triangle (pp. 938, 943). They keep these interparty policy distances constant in order to model the isolated effect of the status quo on government formation (p. 938). The fourth and final major difference is the aforementioned fact that office benefits can be negative (p. 943). Hence, even for a majority formateur party it is possible, given the circumstances specified below, to gain more utility from coalescing with other parties than from forming a single party government, because those coalition partners can offer the formateur benefits in addition to the ones the latter would obtain when governing alone (p. 948).

Taken together, these four unique assumptions cause Baron & Diermeier to arrive at predictions that are substantially different from those of Crombez. First of all, following from the take-it-or-leave it proposal procedure, the formateur always obtains all of the benefits from holding office in addition to extra benefits offered by its coalition partners (p. 943). This is because the formateur always has a strong bargaining position, as the failure to form a government results in the maintenance of the status quo (ibid.). The formateur is therefore in a position to demand rents from the parties in exchange for adopting a policy that is more favorable to them than the status quo (ibid.). For their predictions regarding the size and makeup of governments, Baron & Diermeier explore two scenarios: one in which no party has an absolute majority of the seats, which they call a minority parliament, and one in which one party does have a majority, which they call a majority parliament (ibid.). In the first scenario, a formateur party *i* will normally form a minimal winning government with the party whose ideal point lies furthest away from the status quo, as the disadvantage this party is experiencing from the status quo is translated into a higher payment of office holding benefits to the formateur in exchange for a more favorable policy (ibid.). If the status quo is equally distant from parties *j* and *k*, party *i* will be indifferent as to which party it will form a government with (ibid.). If, on the other hand, the status quo is sufficiently distant from both *j* and *k*'s ideal points[18], party *i* will form an oversized government

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[18] When the parties' ideal points are normalized such that they are located respectively at the points (0, 0), (1, 0) and (½, $\sqrt{\frac{3}{2}}$), Baron & Diermeier derive a minimum distance of $\frac{1}{\sqrt{2}}$ between a parties' ideal point and the status quo q for the boundary condition at which minimal winning coalitions are observed (p. 946). If we let D^i denote a circle with a radius of $\frac{1}{\sqrt{2}}$ around party *i*'s ideal point, we get three intersecting circles D^1, D^2 and D^3 around the three parties' ideal points (pp. 946-947). If q lies inside any of the intersecting areas, such that $q \in D^{12} \cup D^{13} \cup D^{23}$, minimum winning governments form (ibid.). If the status quo lies inside only one of the circles, for instance D^1, and in none of the others, such that $q \in D^1$

with both *j* and *k*, obtaining office benefits from both (p. 948). Turning now to the scenario of a majority parliament, let party 1 be the majority party, which is automatically selected to be the formateur (ibid.). If the status quo lies relatively close to both party 2 and party 3's ideal points, party 1 forms a one-party government (ibid.). If the status quo is close to one party's ideal point but distant from the other party's, party 1 forms an oversized government with the party lying farthest from the status quo (ibid.). If, on the other hand, the status quo is sufficiently distant from both party 2 and 3's (or from all three parties') ideal points, party 1 forms a consensus government with both[19] (ibid.).

In general terms, then, the more outlying the status quo in relation to the parties' policy positions, the larger the governments that form. But what causes a status quo to be outlying for the majority of parties or even all parties, given that previous governments were composed of those same parties and reflected their policy positions? Baron & Diermeier suggest that shocks to the status quo can come in the form of economic or security crises, which explains the frequent real-world observation of national unity cabinets directly after such crises (p. 948).

One implicit assumption underlying this prediction is that, as the number of parties in office increases, the total pie of office benefits grows. Put differently, there is no fixed amount of office holding benefits to be distributed, because in that case, a majority party would not form any coalition, let alone a consensus coalition. It would, after all, strike up all of the office-holding benefits anyway and set government policy at its ideal point. If, on the other hand, the total amount of office benefits is variable, it makes sense even for a majority party to add more parties to the coalition—provided that the status quo is extreme—and increase its office holding benefits even more. While Baron & Diermeier correctly note that consensus governments have often been observed following crisis situations (p. 948, cf. Lijphart, 1984, p. 57), their explanation for this seems far-fetched. It is not clear how parties could bring additional office benefits to the table. The authors mention "board seats on public companies or the national television system and transfers to interest groups and party foundations" (p. 935), but the question remains how substantial these benefits are in comparison to portfolios or policy. Baron & Diermeier also claim that voters are indifferent as to the distribution to these resources (ibid.), but parties passing public money meant for their own foundations to those of other parties could be seen as unethical and it is not hard to imagine such practices leading to damaging scandals.

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and $q \notin D^2 \cup D^3$, party 1, if it is selected as the formateur, proposes a consensus government (ibid.). If, in this same situation, either party 2 or party 3 is selected as the formateur, they form majoritarian, but not consensus governments (ibid.). If q lies outside any of the circles, such that $q \notin D^1 \cup D^2 \cup D^3$, a consensus government is always formed irrespective of the party that is selected as the formateur (ibid.).

[19] The formal definition of this proposition is as follows. If $q \notin D^2 \cup D^3$, party 1 forms a consensus government (p. 949). If $q \in D^2 \cup D^3$ and $q \notin D^2 \cap D^3$, party 1 forms an oversized government with the party *j* that lies farther from the status quo, provided that $q \notin D^j$ (ibid.). If $q \in D^2 \cap D^3$, party 1 forms a single-party government (ibid.).

An even more extreme example would be an all-inclusive war cabinet in which the leading party exploits the situation to extort its coalition partners. It very difficult to imagine this not coming to the surface and sparking substantial outrage. The applicability of Baron & Diermeier to real-world situations therefore appears to be limited.

Theories on the Formation of Legislative Coalitions in the US Congress

In parallel to the theoretical literature on coalition governments in (mostly) European multiparty parliamentary systems, another literature was developing which focused on the formation of legislative coalitions around law bills in the US Congress. Both literatures rely heavily on game theory and take the works of N-M and Riker as their point of departure. However, they evolved in different directions based on their different objects of study. One of the key differences between the two is that, in the parliamentary literature, parties are taken to be the participants of the games, whereas in the lawmaking coalition literature, individual legislators are taken as the participants. Furthermore, whereas one seeks to explain the formation of multiparty coalition *governments*, which are subject to the confidence of the parliament, the other seeks to explain ad-hoc legislative coalitions formed in order to pass laws.

Universalism

Weingast (1979) took as his central puzzle the fact that the minimal winning coalition as predicted by Riker did not match the frequent occurrence of universal or unanimous coalitions around distributive legislation in the US Congress (p. 246). He then set out to explain this seemingly counterintuitive result using a model of his own creation.

Before continuing to this model it is necessary to define the concept of distributive legislation. According to Weingast, “a policy is distributive if the benefits accruing to one area can be varied without affecting the benefits received by other areas” (p. 246). What this means can best be illustrated by an example. In the United States Congress, each legislator is elected by and represents a single district. In order to serve their constituencies and improve their chances of reelection, these legislators seek to obtain funds for, say, public works projects in their districts. For this they must present proposals for such projects to be voted on by Congress. If a legislator gets a project approved, the cost of that project is “spread” across all districts through general taxation (ibid.) rather than taken away from a specific set of districts.

In this context, Weingast compares two different games, the *Distributive Legislative Game* (*DLG*), where bills pass by simple majority, and the *Universalism Legislative Game* (*ULG*), where only “coalitions of the whole” are allowed, and shows that the latter dominates the former in the sense that it provides greater *ex ante* expected benefits to the participants (p. 245).

Given a legislature with *N* members who have preferences over policies and outcomes, the *DLG* is an *n*-person cooperative game[20] in which each representative *i* proposes a project with total benefits *b* and total costs *c*, where the benefits outweigh the costs ($b > c$), that is, the project is net beneficial (p. 247). While the benefits of the project accrue only to legislator *i*'s district, the costs are spread evenly over all districts through the taxation system (ibid.). This taxation mechanism limits the possibilities for side-payments (ibid.), which, as one may recall from the discussion on the minimal winning coalition, are private agreements between a coalition's participants about the division of the coalition's gains. Furthermore, by virtue of the nature of distributive policy, the zero-sum condition does not hold (ibid.). These two conditions distinguish the *DLG* from the simple majority games of N-M and Riker (ibid.).

If a single legislator proposes a project, it will be rejected by the legislature because the payoffs to the districts of the other legislators are negative (taxation is imposed on all districts to pay for the project) (ibid.). Therefore, legislators have to work together to form majority coalitions and pass one another's proposals, a process which is customarily referred to as *logrolling* (ibid.). The *vector valued* characteristic function of this game, meaning the value of a play obtained by a single participant, is given by:

$$v_i(S) = b - \frac{|S|}{N} c \text{ for } i \in S, |S| > \tfrac{1}{2}N$$

meaning for legislators *i* members of a winning coalition *S* with size > ½*N*, and

$$v_i(S) = -\frac{|S|}{N} c \text{ for } i \notin S$$

meaning for legislators that are not members of a winning coalition (p. 247). Hence, a legislator that is a member of a winning coalition gains the benefits *b* of his project and pays his individual share of the cost *c* of that project, which is equal to one-*N*th of |*S*| projects (recall that the costs are spread over *all* districts and not just the districts of the 'losers', the legislators that are not part of the winning coalition, as would be the case in the simple majority game) (ibid.).

Weingast argues that, due to the lack of the zero-sum condition and the fixed distribution of side-payments imposed by the *LDG* taxation mechanism (p. 248), the *LDG* has a more

[20] Since John Nash (1951) introduced the concept of non-cooperative games, game theory has been divided into two main branches: cooperative game theory (CGT) and non-cooperative game theory (NCGT) (Chatain, 2016). While both focus on the same subject matter, each asks different questions. CGT is interested in figuring out how participants, acting cooperatively, can maximize their collective payoffs (ibid.). Concepts like the characteristic function and the core are relevant here (ibid.). Finding outcomes that are 'group rational' is the key idea. On the other hand, noncooperative game theory concerns itself with determining the individual strategies participants should follow in order to maximize their individual payoffs (ibid.). Here, the Nash equilibrium is an important concept (Nash, 1951). This equilibrium ensues when each individual participant follows the most advantageous strategy (ibid.). Hence, individual rationality drives NCGT.

determinate solution than the zero-sum majority games of N-M and Riker, which consist of an infinity of imputations or distributions of side-payments. That is, while the solution to both games is the same minimal winning coalition or *MWC*[21], once an *MWC* forms in the *LDG*, it cannot be overturned by a different coalition (p. 249) with a different distribution of payoffs, as this is fixed by the tax mechanism. This is quite unlike the simple majority games of N-M and Riker, which are characterized by cyclical domination. The set of *MWC*s therefore constitutes the core of the *LDG* (p. 248).

While none of this is quite that revolutionary, Weingast's main argument is that there is another game that dominates the *LDG*, namely the aforementioned *Universalism Legislative Game* or *ULG*. Recall that the main difference between both games is that the *ULG* only allows for coalitions of the whole. This completely changes the payoffs participants can expect *ex ante*. Whereas a participant of the *ULG* can be assured of a payoff to his district which is equal to *b* – *c* (*N* gets cancelled out because each legislator pays one *N*th of *N* projects), a participant of the *LDG* is not guaranteed a place in the *MWC* and will therefore not necessarily get a positive payoff (pp. 250-251). Assuming that each legislator is equally likely to be included in the *MWC*, the probability for this to happen is equal to

$$\frac{(N+1)}{2N}$$

Proof[22]. Let this probability be denoted by *a*. First note that the fraction $\frac{|S|}{N}$ in the characteristic function can be rewritten by substituting |*S*| for $\frac{N+1}{2}$, which is equal to the size of a minimal winning coalition given a legislature with an odd number of members (*N* is odd). $\frac{|S|}{N}$ then becomes $\frac{N+1}{2N}$, which happens to be equal to the probability *a* of *MWC*-membership. Hence, rewriting both parts of the characteristic function in this manner, multiplying each with the

[21] Proof that the *MWC* is the solution to the *LDG*: if there exists a minimal winning coalition *S* and a non-minimal winning coalition *A* that is bigger than *S*, such that |*A*| > |*S*|, then a legislator *i* that is a member of both *A* and *S* (these two coalitions have some overlap because both contain more than half the members of the legislature), such that $i \in A \cap S$, prefers *S* over *A* because the payoff is greater: $b - \frac{|S|}{N}c > b - \frac{|A|}{N}c$ (pp. 248-249).

[22] Let $M = \frac{N+1}{2}$ be the majority threshold (p. 251, footnote 9). The total number of minimal winning coalitions is equal to $\binom{N}{M}$ (ibid.). This function is known as the *binomial coefficient* and represents the number of ways in which *M* objects can be taken from a set of *N* objects **(ref.)**. It is defined as $\binom{N}{M} \coloneqq \frac{N!}{M!(N-M)!}$ **(ref.)**. The total number of minimal winning coalitions of which a given legislator is not a member is equal to $\binom{N-1}{M}$ (Weingast, 1979, p. 251, footnote 9). Hence, the probability for a given legislator to be included in an *MWC* is equal to $1 - \frac{\binom{N-1}{M}}{\binom{N}{M}}$ (ibid.). Calculating this yields $1 - \frac{\frac{(N-1)!}{M!(N-1-M)!}}{\frac{N!}{M!(N-M)!}} = 1 - \frac{N-M}{N} = 1 - \frac{N-M}{N} = \frac{N+1}{2N}$ (ibid.).

respective probability of each outcome and then summing them, we obtain the expected payoff EP_D:

$$\begin{aligned} EP_D &= a(b - ac) + (1 - a)(-ac) \\ &= ab - a^2c - ac + a^2c \\ &= a(b - c) \end{aligned}$$

(p. 252). Recall that the expected payoff under ULG is equal to $b - c$ in all cases, thus $EP_U = b - c$ (ibid.). It can now be demonstrated that $EP_U > EP_D$, because

$$\begin{aligned} EP_U - EP_D &= (b - c) - a(b - c) \\ &= (1 - a)(b - c) > 0 \end{aligned}$$

(ibid.).

This simple calculation shows that, if legislators cannot be certain of winning coalition membership, universalism provides a higher *ex ante* expected payoff than direct majority rule. Weingast argues that the choice between these two games, LDG and ULG, really is a choice between institutions (p. 250). Therefore, rational legislators, when designing their institutions, have good reasons to opt for decision-making by unanimity rather than majority rule (ibid.). Aside from yielding a higher expected payoff, universalism reduces the bargaining costs (time and energy invested) of coalition formation (ibid.): rather than a new *MWC* having to be negotiated from scratch at the start of each round, such as under majority rule, under universalism there is only the coalition of the whole which does not change.

The previous discussion shows that in the long run, legislators are better off under universalism than under majority rule (p. 253). But what about the short-term? A group of short-term-oriented legislators may still propose a minimal winning coalition (ibid.). A punishment mechanism must therefore be instated to make sure legislators do not deviate from the unanimity rule (ibid.). For instance, whenever a legislator proposes an amendment removing one of the items from the universal logroll, a punishment could consist of removing his or her project instead (ibid.). On the other hand, a natural disincentive to violate the universalism norm is the fear of being excluded from future logrolls, which might result from uncooperative behavior (p. 254).

Another interesting issue is the relationship between the costs and benefits of projects. Until now it was assumed that $b > c$. The fact that this is a necessary condition for universalism to be preferred over majority rule can be shown by recalling that for $EP_U > EP_D$, $(1 - a)(b - c) > 0$. Hence, either $a < 1 \wedge b > c$, or $a > 1 \wedge b < c$. As a, the probability of *MWC* membership, is smaller than 1 (the numerator, $N + 1$, is smaller than the denominator, $2N$ for $N > 1$), b must be greater than c (ibid.). However, Weingast shows that even for inefficient legislation, meaning c

> b, universalism can be maintained. After all, once universalism has been instituted along with its enforcing mechanisms, a prisoner's dilemma[23] ensues and legislators, acting individually, will continue to propose ever less efficient legislation until the benefit of a bill drops below its share of the cost for the individual district, such that $b < \frac{1}{N}c$ (pp. 254-255). Another reason inefficient legislation is possible is that the political costs and benefits of legislation differ from the economic costs and benefits, and legislators are mostly interested in serving their constituencies in order to be reelected (p. 255). To do so, legislators do not need to capture the vote of their district's entire electorate, they only need to construe majority voting coalitions (ibid.). In case a legislator has managed to build a constituency representing 60% of his district's electorate, he can serve this constituency by concentrating among it the benefits of his legislation while socializing the costs (pp. 255-256). Hence, he can include a project as long as $b > .6c$ (p. 256).

Nevertheless, even for $\frac{1}{N}c < b < c$, the system eventually becomes electorally unsustainable, and will be abolished or altered by cooperative action (ibid.). Furthermore, if all legislators propose projects with cost-benefit ratios in the aforementioned range, a single legislator can make himself better off by vetoing the whole package (pp. 257-258). In this situation, the payoff to the individual legislator will be net negative, $b - \frac{1}{N} * Nc = b - c < 0$, whereas vetoing the package yields a payoff of zero (p. 258). However, surprisingly enough, more inefficient legislation is possible under majority rule than under universalism. That is because, under the former, $\frac{N+1}{2}$ projects pass, the cost of which is shared by the districts such that each district pays $\frac{1}{N}$ of $\frac{N+1}{2}c$, that is, $\frac{N+1}{2N}c$ (p. 257). The benefits to the district are then positive as long as $b > \frac{N+1}{2N}c$ (ibid.). Even if all members of the *MWC* propose legislation with a cost-benefit ratio approximating this value, this does not constitute an electoral liability for them collectively, because almost 50% of the cost of the legislation is paid by the losers, who do not get to propose projects. In other words, the benefits are concentrated among a bare majority, while the costs are spread over the entire legislature. This is different from universalism, where costs *and* benefits are distributed more or less equally among all legislators.

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[23] The more general meaning of the well-known prisoner's dilemma is a situation in which individuals who have a rational incentive to cooperate nevertheless decide not to do so **(ref.)**. Applied to this situation, it would be more beneficial to the legislators collectively to reduce inefficient legislation, but because each of them individually is able to propose inefficient legislation that is only efficient from the perspective of their district, they continue to do so, while fearing punishment if they propose to cut inefficient projects from the logroll. In the prisoner's dilemma, each prisoner also fears punishment if she decides to cooperate but the other does not.

Committees

If Weingast (1979) demonstrated that it can be rational for long-term oriented legislators engaged in distributive policy making to institute universalism, Fiorina (1981) shows that this is even more so when legislatures are internally differentiated into a number of committees with different policy jurisdictions. However, before bringing in committees, he, like Weingast, explores two scenarios, one characterized by what he calls *unconstrained majority rule* and the other by universalism. Fiorina follows Weingast's conclusions on universalism, but disagrees with him on majority rule: whereas Weingast concludes that the minimal winning coalition is a stable outcome under those circumstances, Fiorina considers the possibility of 'bribe coalitions' in which the losers bribe away the cheapest member of the minimal winning coalition (similar to what we saw in the section on the minimal size coalition) by offering to build only his project (p. 16). This possibility renders the minimal winning coalition vulnerable to such bribing coalitions, which are in turn vulnerable to other bribing coalitions, and so we are left with domination cycles again (pp. 16-17). This ultimately means that the rejection of all projects, the so-called *zero project outcome* Z, is the unique outcome of majority rule (p. 14).

However, Fiorina demonstrates that, with the imposition of a number of simple rules constraining majority rule, stability can be restored (p. 17-21). These rules limit "legislative maneuvers to those which entail marginal or incremental changes from proposals under consideration" (pp. 17-18). In practice, this implies that only small alterations to existing proposals can be made, and no wholly new proposals can be introduced. These alterations or amendments that can be made to omnibuses are limited to the following: "(1) Strike a project[;] (2) Add a project[;] (3) Substitute a project for another" (p. 18). Such constraints are known as a Ψ rule (Luce, 1954, cited by Fiorina, 1981). These rules do not allow for bribe coalitions, which would require proposing a whole new omnibus.

Nevertheless, even under these rules, Z, the status quo, is in equilibrium (pp. 18-19). In order to avoid this outcome, what must happen is that, at the start of the legislative process, a committee within the legislature must be appointed that will formulate an initial omnibus proposal (p. 18). Note that the legislature is still assumed to be undifferentiated at this point and the committee will effectively cease to exist after formulating the initial proposal. In order for this proposal to have any chance to be accepted by the legislature, it must provide benefits to at least a majority of the legislature's members, so $w > m$, where w stands for the number of projects in the proposal and m represents the majority threshold (p. 19).

When this proposal is introduced to the legislature and legislators are allowed to propose amendments in accordance with the previously mentioned rules, what will happen is that, if a legislator proposes to add a new project benefitting his district, it will be rejected by the totality of the remaining members of the legislature, as it does not provide them with any benefits and only increases the tax burden of their districts (ibid.). On the other hand, when a legislator proposes to cut a project from the omnibus, the most logical candidate for this being the most expensive project, it will be accepted by the entire legislature with the exception of the owner of that project (ibid.). This process continues until the number of projects has been reduced to m (p. 20). Now, any proposal to cut any further projects will be rejected because it will reduce the omnibus coalition to a minority of legislators, a losing coalition resulting in Z (ibid.). Legislators can still propose substitution amendments, however, and any such amendment substituting a more expensive project by a cheaper one will pass by the same margin (all minus one to one) as the deletion amendments when w > m (ibid.). Evidently, the whole process results in the cheapest minimal winning coalition M^C, from where no further amendments pass (pp. 20-21). In other words, M^C is in equilibrium (ibid.). Following Shepsle (1979), Fiorina (1981) calls this a *structure induced equilibrium*, in the sense that it is brought about by a set of rules, which stands in contrast to a *preference induced equilibrium*, which results directly from the preferences of the participants irrespective of procedural rules (p. 22). In this game, the preference induced equilibrium would be Z (ibid.).

While the aforementioned discussion shows that M^C is a stable outcome under *procedure-constrained majority rule*, the introduction of committees destroys this stability (p. 30), as will be explained now. Assume a legislature L is partitioned into a set of committees, $C = (C_1, C_2, \ldots, C_C)$ $|C|\geq 3$, $|C_i|< m$, meaning at least three committees with the size of each committee smaller than m, and each committee has the exclusive jurisdiction over a specific distributive policy area (pp. 25-26). The committees make decisions by majority rule (p. 26). As no single committee controls a majority in the legislature, sets of committees have to work together to combine their proposals into a *super-omnibus* (pp. 26-27). This is not as straightforward as it may seem, however, because rules are imposed neither on decision-making within committees, nor on inter-committee bargaining, and so both are characterized by unconstrained majority rule (pp. 27-28). Only the legislature as the committee of the whole is subject to rules. Hence, it is theoretically impossible to predict what proposals will come out committees and how those proposals will be combined into super-omnibuses (p. 28).

However, even if we assume that a winning coalition manages to agree on a super-omnibus that is subsequently brought to the floor of the House, things are still complex. As before, the omnibus will ultimately be whittled down to M^C after a series of amendments (p. 29). However the problem is that said M^C may contain the projects of a minority of some committees (p. 29). The majorities of those committees would then withdraw their support from the super-omnibus proposal under consideration (ibid.). Put differently, the *zero project outcome* Z in which all projects are rejected “is preferred to M^C by a minority, but that minority may include committee majorities whose cooperation is necessary to form M^C" (ibid.). Hence, in order for it to be accepted, the super-omnibus representing M^C must be "composed entirely of committee winning project coalitions" (ibid.).

Fiorina pauses here for a moment and reflects on the fact that, given the complexity of forming a super-omnibus that simultaneously represents an M^C in the legislature as a whole *and* entirely composed of committee winning coalitions, while taking into consideration that inter- *and* intra-coalition bargaining are characterized by unconstrained majority rule, a far more likely outcome than such a committee winning cheapest minimal winning super-omnibus (the length of the term reveals its complexity) is for the system to "simply bog down", resulting in Z (p. 30).

As Weingast demonstrated, even if (constrained) majority rule has a stable outcome, long-term oriented legislators prefer universalism to it. What Fiorina demonstrated per the above discussion is that, in a differentiated legislature, majority rule, even if constrained by rules, is even less attractive, making universalism in such a legislature comparatively even more attractive than in an undifferentiated legislature (pp. 30-31). But how to go about implementing universalism in a differentiated legislature? While universalism can be instituted directly within committees, some other provision is necessary for inter-committee bargaining (pp. 30-31). One such provision could be *reciprocity*. If universalism can be described by the phrase "something for everyone", the concept of reciprocity refers to "mutual non-interference [or] mutual support" between "concerned minorities," such as between congressional committees with different jurisdictions (pp. 2-3). In other words, committees agree to "keep [their] hands off" one another's proposals (p. 32). What this means in practice is for committees to support or at least not oppose other committees' proposals that do not affect their own jurisdictions (p. 3). In a legislature with a committee system, reciprocity is thus a precondition for universalism (p. 32).

Vote Buying

Groseclose & Snyder (1996) assume a legislative assembly in which two vote buyers compete for votes (p. 303). These vote buyers can be presidents, governors, party or committee leaders, and they exchange a political resource, represented as money, for the votes of legislators on law bills (p. 304). While the disbursement of the money is conceptualized as a direct cash flow to legislators, in the real world it would likely be written into law bills in the form of pork (p. 306). Vote buyer *A* seeks to pass a policy *x* whereas vote buyer *B* seeks to maintain the status quo *s* (p. 304). Legislators themselves also have preferences with regard to *x* and *s*, and in order to be persuaded to vote differently from their natural, exogenously determined preferences, they must receive a quantity of money that is worth more to them than their relative preference for *x* over *s* or for *s* over *x* (ibid.). The vote buyers take turns making offers to legislators, and vote buyer *A* moves before vote buyer *B* (p. 304). Groseclose & Snyder demonstrate that, under certain circumstances, it is cheaper for vote buyer A to buy a supermajority than to buy a minimum winning coalition (ibid.). The logic behind this prediction is elegant and simple. Assume for simplicity that all legislators are indifferent between the two proposals. If *A* buys a minimum winning coalition, *B* only needs to buy out one member of the coalition in order to defeat the bill's passage and maintain the status quo (p. 305). As the strength of the coalition is as great as its weakest link, every single one of its members must be paid a bribe a that is greater than W_B, which is defined as the maximum amount vote buyer *B* is willing to spend on maintaining the status quo (ibid.). If *A*, on the other hand, buys an oversized coalition with a surplus membership equal to m, *B* has to distribute their bribe over m legislators, meaning that *A* only needs to pay each coalition member a bribe such that $a \geq W_B / m$ (ibid.). Buying an oversized coalition is therefore cheaper than buying a minimum winning one.

Groseclose & Snyder characterize a number of equilibria for different values of W_B and legislative support in which x wins. The most detailed results are obtained for a situation where the majority of legislators is initially opposed to x and the distribution of their preferences is linear. Here, as either W_B or the initial level of support for x increases, the coalition *A* buys increases in size (pp. 309-310). Extrapolating from this result, Groseclose & Snyder make a number of predictions for coalition size in real-world settings. First, legislative coalition size can differ across policy areas. In highly contested areas, for instance, where there are interest groups on both sides, coalition size may be larger than in policy areas with one interest group (p. 312). Second, they identify uncertainty as an additional factor driving oversized coalitions. They then expect that legislature size negatively impacts uncertainty and coalition size due to "the law of large numbers", while the turnover of legislators positively impacts uncertainty and coalition size (p. 312). Recall that Carrubba & Volden, while making a similar prediction regarding turnover, arrive at the opposite conclusion with respect to legislature size, an increase of which makes legislators more likely to defect from the logroll.

Defections

Carrubba & Volden (2000) start from similar assumptions to Weingast and Fiorina, but go a step further than those authors in fully endogenizing long-term thinking and coalition discipline. They present two models of coalition formation, a simple 'baseline model' and a more complex 'full model'. Starting with the baseline model, the main assumptions are a legislature with N members who seek to maximize their own utility by passing bills (p. 263). The benefit B of each bill—all bills are equal when it comes to cost and benefit—accrues to the individual legislator who proposed it, while the cost C is shared among all legislators in the assembly (ibid.). The game then proceeds in four stages. First, a voting rule is chosen unanimously by the legislature as a whole (p. 263). This voting rule represents the majority threshold for the passage of a bill, which can vary between ½ and 1 times the assembly membership (ibid.). Then, a minimum winning coalition *MWC* of size *VN* is randomly selected by nature out of all assembly members in 'good standing' (ibid.). What this means will be explained shortly. Each coalition member gets to propose a bill, and then, in the third stage, all *VN* bills are voted on one by one in a plenary session (ibid.). This is key, because allowing bills to be combined into single packages would preclude defections from the logroll, which is precisely what Carrubba & Volden seek to model (ibid.). When bills are voted on individually, coalition members have the option to defect from the logroll, usually after their own bill has passed (p. 264). Defecting causes the *MWC* to lose its (qualified) majority, which in turn prevents the remaining bills on the docket from passing and, by extension, from being financed, saving their cost (ibid.). If the first legislator on the logroll, who has the greatest incentive to defect, defects after the passage of their bill, their payoff will be given by the benefit *B* of one bill minus their share of the cost of that bill, which is given by *C/N* (pp. 264-265). If, instead, no one defects from the logroll, at the end of the period each legislator will gain a payoff given by *B* – *VC* (ibid.). Note that legislature size *N* gets cancelled out: the cost of *VN* bills are shared among *N* legislators. Clearly, all else equal, defecting is much more profitable than cooperating. However, defecting has long-term consequences: it causes the legislator to be placed in bad standing and excluded from participation in logrolls for the foreseeable future (ibid.). This punishment is applied in the fourth and final stage of the game, after which it returns to the coalition formation stage (stage 2) (p. 263). If no one defects, this will happen after stage 3 is concluded. Each time the game returns to stage 2, future payoffs are discounted by a value δ, which represents the reduced value of the future. This discount value itself represents exogenous factors such as the time it takes to move from one period to the next and to form a coalition, as well the reelection chances of the legislators (p. 264).

While the defection payoff for a single period is given by *B* – *VC/N*, the payoff for future periods, in which one will be excluded from participation in coalitions, is given by the negative payoff – *VC* of not being a coalition member but still having to pay one's share of the cost of *V* bills. The total defection payoff for a coalition member in a given period is therefore given by the payoff of defecting in the current period, to which is added the discounted, negative payoff for not participating in coalitions in future periods, yielding:

$$B - \frac{C}{N} - \frac{\delta}{1-\delta}(VC)$$

(p. 275). This expression also sheds light on the workings of δ, which can take on any value in the interval [0, 1] (p. 264). When the future is highly certain and highly valued, δ approaches 1 and $\frac{\delta}{1-\delta}$ approaches infinity. In this case, the number of future logrolls approaches infinity and so does the cost of being excluded from participating in them. When the future is highly uncertain and valued very little, δ approaches zero and so does $\frac{\delta}{1-\delta}$. In this case, legislators do not expect to participate in any future logrolls and the cost of being excluded from them is therefore zero.

Turning now to the cooperation payoff, the payoff one gets when one does not defect, recall that for a single period this is given by *B* – *VC*. The payoff for all potential future periods is then given by the probability *V* of being part of the coalition, multiplied by the payoff of cooperating with this coalition, again equal to *B* – *VC*, plus the probability 1 – *V* of not being a coalition member multiplied by the payoff – *VC* of not being a coalition member but still having to pay one's share of the cost of *V* bills. The total payoff of a logroll member in a given period to cooperate is therefore given by the cooperation payoff of that period, to which is added the discounted payoff of participating in all future coalitions, yielding:

$$B - VC + \frac{\delta}{1-\delta}\big(V(B - VC) + (1 - V)(-VC)\big)$$

(p. 275). Here, the term containing δ has the opposite sign from the previous expression. This means that δ always positively affects the first legislator's net incentive to cooperate.

Moving on now to consider how the abovementioned variables affect the incentive for the first logroll member to cooperate, we can say that this member will cooperate as long as their incentive to do so is greater than their incentive to defect (p. 265). Plugging the above expressions into this inequality and solving for δ yields:

$$\delta \geq \delta^* = \frac{VC - C/N}{VB + VC - C/N}$$

where δ* is the minimum δ for which cooperation is sustained (ibid.). This δ* then serves as a proxy for the difficulty of cooperation. It then follows from the equation that assembly size *N*, the cost-benefit ratio of bills *C/B* and voting rule *V* each have a slight positive effect on δ*, which means they negatively impact the ease of cooperation (ibid.). Some of these results make intuitive sense at first consideration. Note, for instance, that *N* is completely absent from the cooperation payoff, as *N* only affects the short term cost that is saved from bringing down the logroll after getting one's bill passed, and is canceled out when cooperation is sustained for entire periods, as explained before. On the other hand, the effects of *V* and *C/B* are harder to grasp at face value as they affect both incentives in complex ways.

However, Carrubba & Volden's finding that is of most interest to the research at hand is their demonstration that cooperation is easier to sustain under oversized coalitions than under minimal winning coalitions (p. 269) and that coalition size is affected by the same variables that affect the ease of cooperation **(p. ?)**. Now, in order to allow for oversized coalitions, Carrubba & Volden present an additional, more complex model, their so-called ‘full model’, in which they change some of their initial assumptions. The main difference with the ‘baseline model’ is the fact that, here, a coalition is not randomly formed by nature, but rather, a proposer is randomly selected out of the entire set of assembly members, not only those in good standing (p. 268). This proposer can then make a proposal for a coalition with a size, membership and bill ordering of his choosing (ibid.). If all prospective coalition members agree to the proposal, the game advances to the bill voting stage (ibid.). If not, a new proposer is selected (ibid.). Defectors are still placed in bad standing, but in principle nothing prevents a proposer to invite a member in bad standing to his desired coalition. This is, however, not without consequences. In fact, it will result in the proposer being placed in bad standing as well (ibid.).

In case an oversized coalition is formed consisting of $S = VN + A$ members, where A denotes the number of surplus members beyond minimal winning size, the defection game changes considerably. Now, a potential defector knows that if they alone defect, the logroll will still have a (qualified) majority and the remainder of the bills will pass (p. 269). Hence, they have to ally with $A + 1$ coalition members and defect as a group (p. 275). But now, for potential subsequent defectors, there is a trade-off: if they don’t go along with the defecting group and decide to cooperate with the coalition, they will have to pay for the remainder of the bills on the docket, but they will also have a greater chance to be invited to future coalitions, because the defecting group will be in bad standing and will be excluded from participation in coalitions (p. 269). The potential last member of the defecting group faces the exact same incentives as the first member of a *MWC* of size *VN*, and therefore, if one wants to determine the minimum surplus membership for which cooperation is sustained, one must derive cooperation and defection payoffs for this member (p. 275, note 29). When these payoffs are then plugged into an inequality and solved for A, one arrives at the following equation:

$$A \geq A^{*} = N - \left(\frac{\delta}{1-\delta}\right)\left(\frac{VN}{VN-1}\right)\left(\frac{BN}{C}\right)$$

where A^* is the minimum A for which cooperation can be sustained (p. 269). The effects of the remaining variables on A^* are the same as those on δ, meaning that the greater the size N of the legislature, the cost-benefit ratio C/B of legislation and the voting rule V, the larger the coalition needs to be for cooperation to be sustained. As such, Carrubba & Volden predict the formation of minimum *necessary* coalitions, coalitions that are just big enough to make cooperation possible, and may or may not be oversized (p. 274). However, coalition size is capped by legislature size, and there is a point at which cooperation is so difficult to sustain that no coalition forms at all (p. 269).

What is interesting about this result is that precisely the attractiveness of coalition participation leads to smaller coalitions. This directly contradicts Strøm, who argues that coalitions are smaller when government participation is *less* attractive.

In a subsequent paper, Volden & Carrubba (2004) extend their model to government formation in parliamentary systems. Adapting the variables from their original model, they identify three variables that affect coalition government size: “the number and diversity of actors in a legislative chamber”, where actors can be individual legislators or parties, and which corresponds to N in the original model, the difficulty of passing legislation, which corresponds to δ, and the cost-benefit ratio of legislation, corresponding to C / B (p. 526). The link between the difficulty of passing legislation and δ may be hard to see at first, but when passing bills is hard, for instance when there are many veto actors (p. 528, cf. Tsebelis 1995), there is little benefit to be had from (future) coalition membership, as the chance one gets their bill passed will be low (p. 526). To measure these variables, Volden & Carrubba use the following indicators: for the number and diversity of actors they use the number of assembly members alongside the number of parties, for the difficulty of legislation, a dummy variable indicating whether, in bicameral systems, the upper and lower house are divided, meaning that the largest party in one is not the largest party in the other, and finally, for the cost-benefit ratio of legislation, they use polarization (the more polarized an assembly, the more costly the other parties’ bills are perceived) and the taxes to GDP ratio (the larger the share of the economy controls, the more money is available for projects and the more beneficial legislation) (pp. 528-529).

Theories on the Formation of Government Coalitions in Presidential Systems

Almost all game theoretical models on political coalitions are based either on government formation in parliamentary systems or on legislative coalitions in the two-party single member district system of the United States. Authors writing in this tradition have, however, not considered a very important political system that, in a way, combines elements of both: multiparty presidential systems. Here, coalition builders are presidents who have to play two games: they have to act like a formateur in a parliamentary regime, building multiparty cabinets, while also having to cobble together legislative coalitions. This is because parties in presidential systems tend to be less disciplined than in parliamentary systems (Carey, 2007), and so building majority cabinet coalitions is often not enough to govern effectively: presidents often have to appeal to individual legislators for support in the legislature.

Just like in parliamentary systems, legislative parties and individual legislators in presidential systems are motivated by a number of different resources, such as policy, office and pork, and coalitions are held together by the glue of these resources. While in the contexts discussed before, the distribution of political resources was decided collectively by the prospective coalition partners, whether parties or individual legislators, in the context of government coalitions in multiparty presidential systems we have an exogenously appointed formateur, the president, in whose hands the power over said resources is largely concentrated. Furthermore, the rule of presidents is not conditional on parliamentary confidence like that of prime ministers, and so multiparty coalitions are de jure not required to govern, even when presidents do not command majority parties[24]. These two facts leave presidents with a lot more freedom than prime ministers to build coalitions as they see fit. This also means that the government coalition building and management game in presidential regimes is much more complex and multivariate than in their parliamentary counterparts.

The Presidential Toolbox

While game theory has not yet been rigorously applied to government coalition formation in multiparty presidential systems, the idea that presidents have to rely on a number of different 'tools' to build stable coalitions is well established in this literature. At first, scholars of coalitional presidential 'regimes' (single quotation marks added because coalition governments in presidential systems are not mandated by institutional design, but are a deliberate decision made by presidents) such as Brazil and Uruguay focused on single resources driving coalition formation, such as the fair distribution of cabinet portfolios among coalition parties (Altman, 2000; Amorim Neto, 2002), the targeted transfer of pork to the constituencies of individual legislators (Ames, 2001) and the formation of 'agenda cartels' in the legislative assembly (Amorim Neto, Cox & McCubbins, 2003). Roughly a decade later, however, students of coalitional presidentialism began to integrate multiple coalition resources into unified models. I will call this genre of scholarship 'toolbox theory', after Raile, Pereira and Power's (2011) term 'presidential toolbox', which refers to the set of coalition building 'tools' that presidents dispose of. Raile et al. combined cabinet portfolios with pork as the two main variables driving coalition maintenance as measured in terms of the month-to-month variation in legislative support enjoyed by the government (p. 325). Importantly, they demonstrated that these two resources are substitutable, meaning that surpluses in one could offset deficits in the other (ibid.). However, each resource played a slightly different role: while cabinet appointments established a 'baseline of exchange', pork expenditures served to maintain legislative support over time (p. 324).

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[24] A caveat exists for president-parliamentary regimes where the cabinet, but not the president, depends on parliamentary confidence. However, because the president can unilaterally dismiss cabinet ministers, he or she is still the dominant actor when it comes to the distribution of portfolios **(cite Shugart & Carey)**

The idea of substitutable coalition resources is of course nothing new, and features prominently in the works of Strøm, Crombez and Baron & Diermeier. Raile et al., however, were the first to explicitly apply this logic to coalition management in presidential systems. They were followed by Chaisty, Cheeseman & Power (2014, 2018), who added legislative agenda power, 'partisan powers' of presidents over their own parties, and the informal exchange of favors to the toolbox, bringing the total to five tools. Amorim Neto (2018) came in from a slightly different angle, focusing exclusively on coalition cabinets rather than legislative coalitions, the latter being defined as the set of legislators on the assembly floor that in fact concede support to the president, whether or not their party is represented in the cabinet (Chaisty et al., 2014, p. 74). Furthermore, Amorim Neto took an ontological rather than an explanatory approach. This means that his aim was not to explain, but to conceptualize majority coalition cabinets, and to define it in terms of a number of secondary dimensions, which partly correspond to coalition management tools in the aforementioned models (Amorim Neto, 2018, p. 295). His main focus is on the legislative arena, to which three out of his five secondary dimensions are related. Tellingly, patronage (office) and pork are combined into a single dimension (ibid.). The other dimensions then are the presence of at least two parties in the cabinet, a purely definitional requirement, the president's strategic use of their agenda-setting powers, party leaders' use of *their* agenda-setting powers, as well as the sharing of the legislative agenda between the president and the coalition partners (ibid.). Note that Amorim Neto includes a non-executive actor, the party leader, in his model.

Details aside, however, the key takeaway from toolbox theory is that constitutional powers enable presidents to traverse the separation of powers and manage coalitions effectively (Cheibub & Limongi, 2010). While this proposition itself does not say anything about the size or makeup of coalitions, in combination with insights from the other models, toolbox theory can help us formulate predictions about those features. This is then the main challenge for future research: building a game theoretical model of coalition formation in multiparty presidential systems that is able to accommodate all of these moving parts while still yielding testable predictions.

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